Interventions that Engaged Teachers in Analyzing Mathematics Classroom Instruction


The Summer Institute

“During the summer of 1999, 22 middle and secondary teachers participated in the integrated science and mathematics institute. Of these 22, 10 had previously participated in an integrated science and mathematics program at WSU. The teacher participants had diverse backgrounds ranging from 2 to 35 years of teaching experience and from no professional development experience to master’s degrees. Additionally, the areas in which the teachers specialized were quite different: four special education teachers, four mathematics and four science high school teachers, and 11 middle school teachers. The three districts involved were a large urban district, a suburban district, and a smaller urban and rural mixture. Student populations ranged from 75% African American students to 85% Caucasian students. These districts were chosen because they are local to the university, and WSU’s preservice teachers are often placed with these districts for field experiences.

The summer institute was intensive with 72 contact hours of class over a 4-week time period, meeting 8 hours a day for 3 days a week. We immersed the teachers in inquiry-based learning environments, in which they worked on integrated science and mathematics units in cooperative groups of three or four. The general structure of the institute involved teachers spending two thirds of each day working on content units and the remainder of the day considering pedagogical issues and developing such units for use in their own classrooms.

Two different cooperative groupings were used. For the science and mathematics investigations, teachers were grouped heterogeneously with the requirements that the teachers in the group could not all teach the same grade level, or be in the same district, or teach the same subjects. For the development of units, cooperative groups were formed by same, or similar, grade-level teachers, since these teachers shared similar curricula and were often from the same district. Including both elementary and secondary science and mathematics teachers within the same groups for content investigations effectively expanded the resources and expertise available to groups in both content and pedagogical knowledge. Rich discussions resulted from these heterogeneous groups, often involving topics of vertical curriculum alignment and effective pedagogical strategies. Heterogeneous grouping typically developed mutual respect and cooperation among the different grade level and topic teachers.

To best model standards-based integrated science and mathematics teaching practices, we team-taught the institutes. In this way, teachers experienced teaching from both the science and mathematics perspectives and gained pedagogical knowledge of both disciplines. Master’s degree program students who were also secondary science and mathematics teachers helped facilitate the institutes. These ‘resource’ teachers provided real classroom connections that aided participants in transferring the institute experiences to the precollege classroom.

Due to the diverse backgrounds, teaching assignments, and teaching environments of the teachers, the content of the institute was matched to grade 4-12 strands of the science and mathematics standards, with topics chosen for their importance and integration aspects. Content investigations started with the most fundamental concepts, usually encountered in the earlier grades, and built up to the concepts and applications of the upper grades. Even though the content was consistent with
grades 4-12 standards, the teacher participants analyzed the content at an adult level in order to develop the conceptual understanding necessary to teach effectively.

We used a combination of commercial curricula and curricula we designed. Commercial resources included *Mathematical Modeling in Our World* (The Consortium for Mathematics and Its Applications, 1998a) and physics education materials (Arons, 1997; McDermott, 1996). The integrated science and mathematics units we designed ourselves were adapted from preservice teacher course activities (Basista, 1998a, 1998b). When designing the units, we took great care to maintain conceptual development for both disciplines. Indeed, we chose many of the specific science and mathematics topics not only for their importance in the teachers’ curricula, but also because the topics lent themselves to a high degree of integration. In every case, we made no assumptions about the backgrounds of the teachers. Each unit started with the most fundamental concepts and built teacher understanding from that basis. Since 1997, we have utilized units such as motion and graphing; shadows and proportional reasoning; and simple machines and proportional reasoning. Refer to Table 1 for the topics covered in 1999.

The integrated science and mathematics units were of a guided discovery format, with facilitator checkpoints included after conceptually connected sections. At the checkpoints, we utilized questioning techniques not only to deepen the teachers’ understanding, but also to model effective questioning strategies. At these checkpoints, we often discussed pedagogical issues related to teaching the material in grades 4-12 classrooms. We assigned daily homework over the sections completed to help solidify the teachers’ understandings of the content and to provide further examples of applications of the concepts.

The pedagogical issues addressed during the institutes related directly to the standards, their implementation, and assessment. These topics included comparisons between inquiry and traditional environments, assessing students’ prior understandings, methods of modifying and/or developing inquiry-based activities, cooperative learning techniques, development of in-depth conceptual understanding, development of problem-solving skills, integration of science and mathematics, reflection on one’s teaching, and authentic assessment techniques. For a sample of pedagogical content covered in the summer institute, see Table 1.

About halfway through the institute, the class was divided into groups of teachers who taught similar grade levels so that they could develop integrated science and mathematics units for use in their classrooms. At this point, the teachers began to apply the science, mathematics, and pedagogical content knowledge they had acquired during the institute to their own classrooms. During the final two days of the institute, the teachers team-taught lessons from their developed units for the class and received peer and instructor feedback.

**Academic Year Support Activities**

We visited the teachers’ classrooms three times during the academic year to observe them, to model teaching methods, and to provide feedback about their teaching practices. During the academic year, the teachers attended three workshops, in which they shared the results of their efforts. During the workshops, pedagogical issues and district issues were frequently discussed. We encouraged teachers to maintain contact with us through phone and email.

Throughout the academic year, the teachers built portfolios documenting their efforts in modifying their teaching practices. These portfolios included lessons they had taught in their classroom,
together with reflections, student feedback, and results. Teachers documented their efforts in implementing inquiry and cooperative teaching practices, developing their students’ in-depth content understanding and problem-solving skills, and utilizing forms of authentic assessment.”

### Table 1
**Summer 1999 Topics**

<table>
<thead>
<tr>
<th>Science Content</th>
<th>Mathematical Content</th>
<th>Pedagogical Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadows</td>
<td>Proportional reasoning</td>
<td>Science and mathematics standards</td>
</tr>
<tr>
<td>Measuring heights and distances</td>
<td>Geometry</td>
<td>Inquiry</td>
</tr>
<tr>
<td>Levers</td>
<td>Multiple representations (graphs, diagrams, symbols)</td>
<td>Integration of science with mathematics</td>
</tr>
<tr>
<td>Hooke’s Law</td>
<td>Logistics/modeling</td>
<td>Developing problem solving skills</td>
</tr>
<tr>
<td>Population growth</td>
<td>Modeling vs. problem-solving</td>
<td>Authentic/alternative assessment</td>
</tr>
<tr>
<td>Population growth</td>
<td>Exponential growth</td>
<td>Cooperative learning</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modifying and developing inquiry lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Questioning techniques</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflective teaching practices</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Facilitating inquiry lessons</td>
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THE STUDY CONTEXT
“Data have been gathered in four research sites in the U.S.: two groups of teachers on the east coast who participated in Fostering Algebraic Thinking (AT) PD and two west coast districts participating in Video Cases for Mathematics Professional Development (VCM) PD. All of the groups were facilitated by the lead authors of the respective programs, (Driscoll for the AT groups and Seago for the VCM groups), ensuring high fidelity of implementation. Both seminars involved 12, three-hour sessions. Both PD programs involved 36 hours of PD (12, three-hour sessions). The VCM groups completed all 12 sessions in a single academic year (2003-2004) and the AT groups completed the PD over the course of three semesters (October 2003- January 2004). In all, 49 middle and high school teachers participated in the groups, 20 in the AT groups and 33 in the VCM groups. Sixteen teachers (four from each site) are being followed more closely to create case studies. Seminar participants included both veteran and early career teachers. Slightly more than a quarter of the teachers participating in the PD had been in the classroom for 5 years or fewer; the entire group of seminar participants averaged approximately 10 years of teaching. In addition to the 49 PD participants, 25 teachers served as a comparison group for our pre/post written measures. These comparison teachers came from the same districts as the PD participants.”

“The study reported here was initiated by the researcher, who is a university professor and a curriculum developer, as a response to a need for mathematics curriculum reform. The researcher initiated a three-year research project that was designed to develop cases with a school-based collaborative action research approach to assist teachers in implementing the spirit of curriculum standards into classroom practices and to reduce the gap between theory and practice. That is, the goals of the research were:

1. to enhance the rethinking of mathematics teaching in classrooms in the spirit of the curriculum standards;
2. to foster teachers’ awareness of children’s learning;
3. to support teachers as they began to put into practice their new vision of a learner-centred approach to teaching mathematics; and
4. to promote teachers’ ability to reflect on their teaching experiences.

The study reported here was designed to examine the effects of constructing cases with a collaborative research team in order to develop knowledge central to teaching. Cases, here, are accounts of episodes from classroom teaching that were found to raise issues or dilemmas for the teachers. The cases were constructed from observing participant teachers’ teaching, discussing the pedagogical issues arising from the observations, and writing the cases into a narrative form…

Teachers were invited to participate in the process of experiencing practical teaching, reflecting on their experiences in weekly meetings, and developing their own insights into teaching through the interaction between personal reflection and theoretical notions offered by other teachers of the collaborative team. Teachers involved in the teacher education program took the role not only of teachers but also that of learners…

CONTEXT OF THE STUDY

*Setting and Participants*

To answer the research questions, the researcher initiated a three-year research project funded by a grant contract from the National Science Council of Taiwan. An aim of the research was to encourage the participants to implement what they learned from the research without losing the authentic context of practice. A school, Din-Pu, with about 780 students and 36 staff, was selected to take part in this study. The school was selected because the researcher was invited by the school as a consultant in learner-oriented mathematics teaching in which the school was undergoing a school-based project for the purpose of solving problems related to the reformed curriculum. Further, the teachers of the school were willing to learn and there was support for the research from the principal and administrators.

Thus, the three-year research project was integrated into the school's project with an action-oriented approach to provide the teachers with opportunities for examining their classroom practice by way of collegial sharing and critical reflection. Each teacher of the team took
responsibility for planning, practicing, and modifying the processes of the research, but the researcher was the pilot of the study and worked with the team to discuss teachers’ implementation needs before the research started. The researcher was expected to contribute more theory than practice, while the four teachers were expected to share more classroom experiences. The researcher acted as a partner to the teachers in helping them put ideas generated in discussion into practice.

The four teachers represent distinct experiences of teaching and academic backgrounds. The name, age and years of teaching for each teacher are respectively; Huei (32, 8), Jong (45, 12), Ling (38, 14), and Sue (34, 13). Huei and Jong graduated from the Teachers College, while the others graduated from universities with teacher education programs. Jong is the only one with a masters degree and she is a consultant for assisting school teachers in Hsin-Chu city with implementing a standards-oriented curriculum to classroom practices. The teachers were selected from the staff of teachers who were teaching in the first grade because they were willing to learn and because they were using the mandate reformed curriculum emphasizing a learner-oriented approach (Ministry of Education of Taiwan, 1993)…

This study was based on providing teachers with the opportunities for dialogue on critical pedagogical issues related to the mandated curriculum. It was therefore necessary to create an environment for teachers’ learning in which, through professional dialogues, teachers could communicate what they were learning in their own classrooms to their colleagues. A professional collaborative team was set up to discuss the situations that occurred in a particular teacher’s classroom and to compare them to others.

The first-grade classrooms were the primary contexts for these teachers to frame problems, analyze situations, and argue the advantages and disadvantages of various ways of teaching. In addition, the contexts of teachers’ learning included participation in regular weekly meetings. There were two reasons for selecting teachers from the same grade to participate in this study. The first reason was that the participants, teaching the same mathematics topics, confronted similar pedagogical problems. Similar mathematical content lent itself readily as a focal point when the teachers met together after observing each other’s lessons to address issues and solve pedagogical problems. Secondly, similar pedagogical issues addressed in the regular meetings drew attention and concern from each participant, leading to in-depth discussions.”
Professional Development Structure and Content
The program consisted of three modules. Each targeting a key content area of the California sixth-grade curriculum (i.e., fractions, ratios and proportions, and expressions and equations) and the respective core concepts. Teachers met face to face at the district office in groups of 8 to 10 led by one or both of two facilitators. Each having a strong background in mathematics and several years of teaching experience. Each teacher was provided with a laptop computer connected to the Internet. Video-based analyses were supported by Visibility, a multimedia platform developed by LessonLab. Each module was structured into three main folders: (a) Content Exploration, (b) Lesson Analysis, and (c) Link to Practice. Within each folder numbered pages guided teachers through a series of video-based analysis tasks each containing a few questions for them to answer. Teachers were asked to watch preselected video segments and type their responses to the analysis questions into a textbox. Some questions also required teachers to point the reader's attention to specific moments of the video. A feature of the software allowed them to click on a button that inserted in their text a time stamp corresponding to a moment of the video they had chosen. Written responses were saved on a server accessible by both the facilitator during the professional development sessions and the researchers, who later were able to analyze them.

Independent work at the computer was interspersed with whole-group discussions led by the facilitator, who sometimes projected participants' written responses onto a big screen. Teachers spent roughly half of the time working independently at their computers and half of time working and discussing as a whole group. After an initial introductory meeting, teachers met six times throughout the school year and spent 2 full days working on each module. Content Exploration and Lesson Analysis were each addressed for a full day, usually a week apart. A teaching window followed, after which teachers met at their school sites for 1 hour to share their teaching experiences as part of the Link to Practice phase. Once a module was completed, they moved onto the next phase. The modules were distributed across the school year so that teachers would participate in the professional development sessions on a particular topic area immediately before they were to teach it to their students. Finally, an end-of-year meeting concluded the professional development program. This was mainly involved at the collection of research data. The following is a description of the content and structure of each folder:

Content Exploration
Content Exploration was aimed at deepening teachers' understanding of core mathematics concepts and was accomplished through a combination of written documents and video. Instead of watching a videotaped classroom lesson, here teachers were exposed through video to a mathematics-focused discussion among other teachers led by a mathematics educator. This had the dual purpose of (a) providing a dynamic setting for teachers to learn mathematics concepts that would make the task more engaging than simply reading mathematics content documents and (b) creating an atmosphere in which teachers feel comfortable sharing doubts they themselves may have on mathematics concepts. In the videotaped discussion, in fact, the
mathematics educator often highlighted inconsistencies and misconceptions in teachers’
mathematical ideas, making it easier for the participating teachers to share their own difficulties.

Teachers participating in the professional development program watched selected segments of
the videotaped discussion and posted online answers to questions aimed at fostering their
conceptual understanding. Individual teachers' responses were then shared in a group discussion
led by a facilitator. Occasionally, concrete materials were provided to teachers for them to engage in the same activities in which the videotaped teachers engaged. A list of concepts that
were targeted in the ratio and proportion module and of questions (organized in what we call ‘tasks’) that were posed to teachers online follows in Table I. For brevity, I list there only the
title of each question. For a few tasks, questions are presented in their entirety later in the
summary of findings. Complete questions for each module can be requested from the author. The
fraction and expression and equation modules followed the same structure.

Lesson Analysis
For each module, the second day of professional development was dedicated to Lesson Analysis.
On this day, teachers began by solving a rich problem that made use of one or more of the core
concepts studied the prior day. They then studied a lesson plan that incorporated the rich problem
and watched the video of the lesson in which the rich problem was taught. This videotaped
lesson provided teachers with a model for engaging students in conceptual thinking. Lessons
were filmed in the teachers' district: thus, the students portrayed in the video were from the same
population of the participating teachers' students. During this phase, teachers answered a series
of questions aimed at the analysis of students' learning and understanding as evidenced in the
video and in samples of students' work. At the end of the second day, teachers discussed ways
the lesson plan could be improved and proposed modifications before going back to their
classrooms to teach the resulting lesson to their students.

Following in Table 2 are the problems taught in the lesson videotaped for the ratio and
proportion module and the titles of the questions that guided teachers' analysis. Similar questions
were asked in the other two modules. As for the content exploration tasks, full text of sample
questions is presented later and the full version of the questions can be requested from the
author.

Table I
Content Exploration-Ratio and Proportion Module
Target ratio and proportion core concepts: Ratio as comparison by division: constant
ratios/constant multiples: proportional relationships.

Task 1: Exploring ratio and proportion
Brainstorm about ratio and proportion
Comment on what teachers in videotaped professional development session say about the
definition of ratio and proportion

Task 2: Ratios as comparisons
Give examples of real-life situations involving ratios
Analyze example provided by teachers in the videotaped professional development session

Task 3: Examining equal ratios
Solve a problem about finding a missing value
Analyze solution methods provided by teachers in the videotaped professional development session

Explain a common student mistake with this kind of problem

Task 4: Solving proportions
1. Discuss the cross-multiplying procedure for solving proportions

Task 5: Issues in teaching
1. Reflect on problems on proportional reasoning without numbers
2. What do students gain from activities such as these?

Task 6: Ratio tables
1. Describe understanding students need to set up and complete a ratio table
2. Describe how you might help students use the properties of cross-products in proportions while continuing to develop proportional reasoning

Table 2
Lesson Analysis-Ratio and Proportion Module

Focus problem in videotaped lesson: Your class has been asked to organize a project called ‘Holding Hands Across L.A.’ You will need to have people line up and hold hands from Downtown to Long Beach (a distance of 87,000 ft.). Your job is to determine how many people you will need to form the line.

Task 1: Analyze the main problem
Solve the problem
Predict students' strategies
Describe prerequisite knowledge and skills
Describe opportunities for student growth or misunderstanding

Task 2: Lesson plan and lesson goals
Make hypotheses on how the lesson will help students achieve the goals
Predict students' responses
Collect evidence of students' learning

Task 3: Verify hypotheses and predictions
Analysis of classroom video
Analysis of student work
Evidence of student understanding
Analyze students' opportunities (and missed opportunities) to deepen understanding of core concepts

Task 4: Lesson improvement
Discuss ways lesson can be improved and adapted to the specific needs of each teacher's classroom

Link to Practice
During this phase, teachers taught the lesson they had analyzed and anticipated in a facilitator-led 1-hr. meeting at their school sites. This phase was aimed at facilitating the application of what was learned during the professional development sessions to teachers' daily practices. At the meeting, teachers were asked to share with their colleagues samples of student work from the lesson they had taught. They were told to select work that represented a range of student performance. Each person took a turn and summarized his or her experience teaching the lesson.
and shared student work to discuss both aspects of the lesson that went as planned and aspects that did not….

Difficulties With Questions That Built on Teacher Knowledge of Their Students' Understanding

During the content exploration days, teachers also found particularly challenging questions that required an analysis of their students' understanding and consideration of ways in which that could be facilitated. Following is one of these types of questions included in the expression and equations module (including the transcript of the video clips teachers were asked to watch) and a few examples of teachers' responses:

The following segments show Dr. B. posing a question, and participants explaining their solution methods. The participants used three methods: guess and check, write expressions, and write an equation. Which of these methods are your students likely to use on their own and which would you need to be prepared to present? If your class were to discuss this problem, what aspects of individual methods or the relation between methods would you want to highlight?

Following is the transcript of the video clips teachers were asked to watch (M indicates the math educator who led the videotaped professional development session, T a teacher in the videotaped session).

**Clip 1**
M: Can you tell me the number I'm thinking of when I multiply it by 2 and then add 3. I get the same answer as when I multiply it by 3 and then subtract 2? So I want everybody to try to figure out—do I have to read it—say it again? Everyone have it? All right, so first work it, see if you can come up with what number I'm thinking of and then once you've done it, talk to the people at your table and see, did they get the same one and how did they come up with it ... How did we do it? Wait—see if you—see if you did it the same way. Okay?
T: (inaudible)
M: Well, we're talking about that, so (inaudible) ....Does everybody have it?
T: Yeah.
M: All right. We'll take a minute. Okay. All right. So who'd like to share the method that was done at your table—yeah, what'd you do?
T: My method was, uh, guess and check. I started with 4 first and then I used 5.

**Clip 2**
M: You said, 2x plus 3 equals 3x minus 2.
T: Mm-hm.
M: Then what'd you do?
T: Okay, so, then I began to move around, um the—as I tell my students—the letters so for instance—2x—I'm gonna subtract 2x on both sides to get rid of—not get rid of—but to choose—to move the 2x so that it's on one side of the equation. So I subtract 2x from both sides and I receive—
M: You're gonna take 2x plus 3 and subtract 2x from that side.
T: Is equal to 3x.
M: 3x.
T: Minus 2.
M: Minus 2. Subtract 2x from that side.
T: All right. And so now I have 3 is equal to 3x—
M: Simplify this you get 3—
T: Right. Three is equal to 3x—I'm sorry, um—1x, uh, minus 2. Now, I add 2 on both sides, because I now want to move that negative 2 onto the other side of the equation sign. So I add 2, so I have 3 plus 2 is equal to x minus 2, plus 2. And then result is 5 is equal to x.
M: And what does that tell you?
T: That tells me that the answer set is—for x is 5.
M: Okay. Now, if you recognize that up here, say well, let's see, three equal sums and take away 2, it must be 5.
M: Do you have to do this step? What do you think?
T: Depends on the teacher.
M: So, so, because- let's face it-part of school- a big part of school- is pleasing the teacher or doing it the teacher's way.
T: Or the test.
M: Or the test way. Exactly right. I mean—but the reality is, if you're trying to find out what's the solution set for this, the idea is when—all you're doing is making equivalent equations. You want to get to a point where you say, ‘Oh, I know what will make this true, 5. It's obvious.’ Well, someone may say. ‘Well. I can see 5 is gonna work here.’ All right. I mean, that's okay. I mean, I would think, too, that's okay, as long as you realize 5 because you know 5 minus 2 is 3. Alright. If, on the other hand, I'm teaching, I want you to show me ways to make something equivalent to get down to this that looks like a letter along side of the number- then that's a different skill that I'm teaching. Alright? Alright. So you know there's only one solution to this equation, which means there's only one solution to that equation….

Year 2 Improvements
To address the difficulties teachers encountered, we introduced four changes in the modules for the second year of implementation: (a) increased specificity of content-related questions, (b) focus on common students' misconceptions, (c) refinement of facilitators' planning and variation in professional development discourse structure, and (d) increased guidance in the analysis of student thinking, I now describe each change in detail.

Increased Specificity of Content-Related Questions
Teacher difficulties with questions that relied on basic conceptual understanding of target mathematics topics were addressed by increasing the specificity of the content-focused questions asked in the online tasks. During the 1st year of implementation, we learned a lot about teachers' difficulties with particular mathematical ideas. This knowledge we developed about our own learners informed the design of the questions in Year 2. These became more targeted and often made explicit connections to other related ideas and concepts to foster teachers' conceptual understanding. An example from the ratio and proportion module illustrates these changes. At the beginning of the module, teachers were asked to revisit concepts of ratio and proportion. In Year 1, two broad questions elicited teachers' knowledge of these topics:
1. Dr. B. [the instructor in the videotaped professional development session] asks participants to write down what they think of when they hear the terms ratio and proportion. Watch this video clip to see him pose this initial question to the participants. In the space below, write what you
think of regarding (a) ratio and (b) proportion. You may use definitions, examples, word
associations, etc.

2. In the professional development session, Dr. B. asks the participants to list what comes to
mind when they hear the term *ratio* and what comes to mind when they hear the term *proportion*.
Click the following link to watch the opinion of the session in which participants share their
responses. As you watch the video, click the ‘quick mark’ button to mark each point you find
especially interesting. This will create a time code link. Follow these steps [a series of specific
instructions follow to post a response explaining what you found interesting about at least two of
the points you marked.

This introductory task was modified in Year 2 to include questions that expanded the definition
of ratio to include the related concept of fraction and explicitly asked to identify ratio and
proportion-related topics in the curriculum. Although the first question remained similar to Year
1, the subsequent questions were modified as follows:

2. A participant mentions fraction as something he thinks of when he thinks of the word *ratio.*
Reference is made to fractions several times throughout the professional development session. Is
there a difference between fractions and ratios’? Use examples to clarify your response.

3. A goal of mathematics instruction is to present mathematics as a coherent and connected set of
ideas. With this goal in mind, answer the following questions: In what other areas of the
mathematics curriculum do ratios and proportions appear? How are these curriculum areas
related to ratio and proportion?

*Focus on Common Students’ Misconceptions*

To assist teachers with the analysis of students' understanding of mathematical idea, we added
questions specifically targeted at the analysis of common students' misconceptions. In some
cases, we provided samples of students' thinking or solutions to problems and asked teachers to
analyze them and to discuss how specific instructional strategies may help students overcome
their misconceptions. In the expression and equation module for example, the following
question was included:

Below is an explanation given by II student of a step used in solving an equation using addition
and subtraction: ‘if you had a positive number on one side of the equal sign, you switch to the
other side, it's gonna become negative, but it's the same thing.’ What does this student
understand or not understand about equality and solving equations? What would a teacher say to
the student or ask the student to clarify her thinking?

In other instances, we asked teachers to predict difficulties that students commonly have with
specific math problems:

Consider the difficulties students may have in solving ‘$7 + 4 = x + 5$. What value of $x$ makes the
equation true?’ What do students need to know about equality to arrive at the correct answer?
What are two incorrect answers students might give, and what misconceptions lead to those
mistakes?
Refinement of Facilitators' Planning and Variation of Professional Development Discourse Structure

To address difficulties with both questions focused on content and on student thinking, we also introduced changes in the facilitators' planning of each professional development session. The researchers and facilitators met during the summer prior to the 2nd year of implementation to identify the main ideas and concepts we wanted teachers to understand at the end of each discussion that followed the individual completion of an online task. This detailed outline of learning outcomes for the participating teachers assisted the facilitators in leading the discussion and in funneling teachers' comments toward better understanding of the target mathematical concepts and of ways students can be assisted in developing more sophisticated understandings. Finally, an intermediate step was built in for some of the most difficult questions. Teachers were asked to discuss in pairs their individual answers before they shared them with the larger group. This was intended to facilitate understanding by providing opportunities to bounce back ideas with one another before the intervention of the facilitator in the larger group.

Increased Guidance in the Analysis of Student Learning

Teachers' difficulties with the analysis of student thinking and learning were addressed by providing increased guidance and more targeted questions that focused on specific classroom instances. In the Year 1 modules, teachers were asked to browse through the lesson video and then assess the effectiveness of the lesson within a single task. The task required them to watch segments that portrayed students working on the main problem and to analyze samples of students' written work. In Year 2, the lesson was broken down in smaller segments and several tasks guided teachers through the analysis of the entire lesson with more specific questions on students' reasoning and teacher's instructional decisions. Year 2 questions assisted teachers in focusing on particular aspects of students' thinking and in analyzing teachers' actions in terms of their effects on student learning.

The following examples illustrate the main differences between Year 1 and Year 2 questions. In the Year 1 ratio and proportion module, the video-based question read as follows:

View the following video segments and verify whether the stated learning goals were achieved. Did the students understand that ratios are multiplicative comparisons? Did the students understand that proportions are equivalent ratios? Did students understand how to set up and solve a proportion with a variable? Have students learned to use various strategies to solve problems involving proportions? Please cite evidence from the video (by marking specific moments of the video) to answer these questions.

In Year 2, teachers were asked to comment on various phases of the videotaped lesson through a series of specific questions. The following are the first few questions that were included in the revised version or the module:

Comment on the beginning of the lesson: What information about student understanding did the teacher get by asking... Describe the solutions generated by students. What does the teacher do while students are discussing their solutions? What role does the teacher take in the presentation of ideas?
How does the classroom discussion promote the development of proportional reasoning? The teacher has prepared counterexamples to test conjectures she had anticipated that students would make. How important do you think this portion of the class discussion is to student understanding? If you would have handled this differently, explain what you would have done and why.

In addition, the analysis of samples or student work was modified to include more examples of student work and an activity that asked teachers to classify student work in terms of level of sophistication or mathematical reasoning as evidenced by the solution strategies used by the students. This activity gave teachers the opportunity to learn to go beyond the surface when assessing students' performance and understanding.

“A brief summary of the seminars over the 2-year period is provided so that the readers will have some understanding of the mathematical content to which the teachers were exposed and how they responded to the content.

An introductory meeting in the spring of 1992 was intended to acquaint the teachers with one another and to give them an indication of what to expect from the seminars, and to give us an indication of what to expect from the teachers. During the session, the teachers were given an excerpt from a lesson on fractions (Borko et al., 1992). In the ensuing discussion, one teacher said:

> Maybe I’m way off the wall, but I don’t teach kids to flip numbers upside down.... So we review multiplying fractions....Then I put up a problem with division. (The teacher wrote a division problem on the board and drew two large Xs through the fractions while reciting the following.) I say, ‘Follow these lines and multiply, and you got your answer. Just go ‘I hate math; I hate math. Boy! Do I really hate math!’ (See chapter 8 for more detail.)

The other teachers responded positively to this method. They felt that teaching fractions was extremely difficult; any ‘gimmicks’ would be useful. They indicated that they did not think it was possible to ‘teach fractions with understanding,’ and some also used Explorer calculators for multiplying and dividing fractions. They felt that students’ demands for the answers prevented them from teaching conceptually.

**Year 1.** In the fall of 1992 we held 2 full-day seminars for the teachers. We began by discussing data on how children compare decimal numbers (from Resnick et al., 1989), then worked on place value with decimal numbers via the Blocks Microworld (P. Thompson, 1992). In the afternoon of the first day the teachers completed the Content Knowledge Assessment instrument. We made copies of the completed tests and discussed them among ourselves in terms of the areas on which we should focus our efforts. The tests were returned to the teachers unmarked, and the second day was devoted to discussing the items on that test. The teachers became very involved in considering their own responses and those of the others, then thinking about how their students would react to some of the items. Some of the items had been used with students (Armstrong & Larson, 1995), and the ways in which students thought about those items and solved them were discussed with the teachers. (When relevant, each teacher’s work on this assessment is discussed in the individual case studies.)

For the remainder of the year, approximately half the seminars were presentation-focused—that is, a researcher prepared a presentation based on research with children. The presentations were informal, and there were questions and discussions throughout the presentations. When the presentations were made by visiting researchers from outside the university, a few additional teachers were invited, so that the audience was approximately a dozen. (The presentation-based seminars were substantially the same as the written versions of the presentations appearing in *Providing a Foundation for Teaching Mathematics in the Middle Grades* [J. Sowder &
Schappelle, 1995] as chapters by Armstrong & Bezuk, Harel, Kieren, Lamon, Mack, J. Sowder, L. Sowder, and P. Thompson.) The remaining seminars focused on follow-up discussions of these presentations, on discussions on topics selected by the investigators on the bases of their knowledge of the teachers’ content understanding, of results of tests and interviews of the students of the teachers, and of questions raised by the teachers. (A more detailed presentation of the teacher interactions and struggles to understand the content of these seminars is presented in chapter 4 and also in J. Sowder & Philipp, 1995.)

The first two seminars were intended to lead the teachers to see the value of sense-making as part of the enterprise of teaching. A presentation on rational number sense led to practice with mental computation and estimation and to examination of sense-making with operations and algorithms.

The next four seminars focused on developing the teacher’s understanding of fractions and fraction operations. Presentations by Mack, Armstrong, Bezuk, and Kieren provided the teachers with research-based ways of presenting critical ideas about fractions and fraction operations. Examples of students’ thinking and working with fractions challenged the teachers to think about their role in teaching fractions in meaningful ways. The fourth seminar was devoted to discussion of the results of the teachers’ students’ work on the Fraction Understanding Test (provided in Appendix F). The items tested for conceptual understanding rather than algorithmic skill. The teachers were surprised and distressed with the results. Although they recognized that they were not responsible for the poor performance (the tests were administered after students had been in their classes for less than 2 months), they also realized that until this seminar they had little comprehension of what their students knew and did not know, thus making it difficult for them to base instruction on students’ knowledge. (This seminar was summarized and analyzed in Armstrong, Philipp, & J. Sowder, 1993.)

A more holistic look at both whole number and rational number operations was the subject of the presentation ‘Addressing the Story-Problem Problem’ by L. Sowder. He discussed the connections between the operations and the real-world applications, focusing on what elements in a situation lead to choosing the correct operation.

The next three seminars were informal; they focused on critical incidents in the teachers’ own classrooms and on discussion of the previous presentations. The teachers compared ways that their own planning for instruction on fractions was changing.

In the two following seminars we turned to the topic of proportional reasoning; the discussion was based on a presentation by Lamon. Proportional reasoning as multiplicative reasoning was discussed in some detail. These seminars led into Harel’s presentation in which he outlined students’ progress through additive reasoning into multiplicative reasoning. The final presentation of the year, by P. Thompson, focused on quantitative reasoning in both simple and complex situations.

For the closing seminar of the first year we chose several transcript excerpts from the seminars during which teachers had struggled with mathematical concepts and had finally came to a deep understanding of them. The teachers were given the assignment of reading excerpts and providing written reactions to them at a later date. To set the stage for this assignment, the
investigators each earlier wrote reflections on the year’s work and shared them with the teachers at this seminar. For the remainder of the seminar, the teachers talked informally about what they had learned and how they had changed over the course of the year. The conversations focused on the seminars, our classroom observations, their own planning and insights, and their classroom interactions with students.

**Year 2.** During Year 2, several of the topics introduced during the Year-1 seminars were revisited, sometimes through discussions of (sometimes videotaped) segments of the participating teachers’ classroom rational number lessons that had been observed by the researchers and sometimes through revising the papers written by the presenters of Year 1. The first seminar of the year was devoted to eliciting individual teacher reflections, partly to determine ways to provide seminars of most benefit to the teachers at this stage in their participation in the project. Teachers spoke about their mathematical goals for the year, their mathematical expectations for their students for the year, their roles as teachers, perceived obstacles in teaching mathematics, the growth of students from additive to multiplicative reasoning, changes they were making or would like to make in their mathematics teaching, and what each hoped to gain from the project during the coming year.

In seminar discussions of observed teachers’ classroom lessons (sometimes with videotaped segments presented), the importance of consistently relating the part to the unit was an issue in both the fractions and decimal lessons being discussed; all of the teachers recognized this as an issue in their own classrooms. Issues related to the use of models for rational numbers also arose in these seminars. Before one seminar, two researchers had visited the same teacher a few days apart; the second had the opportunity to see implemented the first’s suggestion to incorporate proportional reasoning into a lesson. Describing this lesson sparked a discussion of teachable moments—awareness of situations in which opportunities to develop important ideas, in this case proportional reasoning, arise.

The one topic tested on the initial Content Knowledge Test but not addressed during Year 1 was that of weighted average in rate problems. This difficult topic was approached in Year 2 through the use of P. Thompson’s *Over and Back* (1994) microworld.

Just as in Year 1, most of one seminar was devoted to discussion of students’ fraction-understanding-test and interview results. The teachers appreciated the limitations of the pencil-and-paper instrument, even though it focused on conceptual learning and the greater richness of the responses in interviews in which answers could be probed for reasoning and in which misinterpretations of the problems were evident.

During these Year-2 seminars, even more than in the Year-1 meetings, the teachers often raised questions or shared classroom experiences that led to extended discussions (e.g., Darota gave students a problem to do individually so that she could work on report cards, but the problem instead turned into an extended lesson on ratio). Issues about standardized tests and textbooks were raised repeatedly. The importance of the teachers’ having deep understanding of the content, the big ideas within a topic, the connections among topics—instead of merely presenting interesting problems that are not necessarily part of a bigger, overall picture of rational
numbers—was recognized by the teachers and was raised by them more than once during the Year-2 seminars.

Year 2 concluded with a seminar in which we, the researchers, explained that we would now be trying to tell what had been learned from this project, and the teachers were asked to reflect on their participation and to tell what had been learned from their points of view. The teachers spoke quite passionately about how much they had learned and about the need for all teachers to have more opportunities to focus on mathematics during professional development.”


(a)

Participants in the Intervention
“The subjects were 48 middle-grades (4-8) teachers participating in Project LINCS. These participants, all volunteers, came from 30 schools and 12 school districts within a 50-mile radius of a Midwestern university. Pairs of participants from the same school were actively sought, but only 32 came from schools with 2 or more participants. Their mean number of years of teaching experience was 13.6….

Intervention Program
Project LINCS was a 3-year intervention program designed to enhance teachers’ knowledge through annual 4-week summer content courses, accompanying 8-hour research seminars on student cognition, and 6 half-day seminars during the academic year focusing on pedagogical practice. The intervention also incorporated structured on-going teacher collaboration and reflection.

Content courses. The summer content courses addressed probability and statistics in year 1, geometry in year 2, and algebra in year 3. The probability and statistics course emphasized the exploration of data and the use of simulation to determine probabilities. Visual displays and descriptive statistics were used to examine characteristics and patterns in data; and theoretical probabilities, simulations, and data analysis were used to solve a wide variety of probability problems. The geometry course focused on the exploration of two- and three-dimensional shapes using the van Hiele (1959/1985) levels of recognition, analysis, and informal deduction as a basis for instruction. In particular, the course incorporated an investigation of polygons and their properties; tessellations; polyhedra and their properties; length, area, and volume measures; and motion geometry. The algebra course explored families of functions in problem contexts. This exploration used graphical, tabular, and symbolic representations to investigate linear, quadratic, and exponential functions. The instructional approach adopted in each course can be described as ‘teaching via problem solving’ (Schroeder & Lester, 1989) and modeled the pedagogy advocated in the half-day seminars. Computers and graphics calculators were used in all courses.

Research seminars on student cognition. The companion research seminar reviewed and discussed research findings on students’ cognition in each of the three content areas and reflected on the implications of these for classroom instruction. The seminars examined the research on the development of probabilistic thinking (Shaughnessy, 1992); van Hiele levels (Fuys, Geddes. & Tischler, 1988); and students’ understanding of variables and their uses (Kieran & Chalouth, 1993). Each year, participants also interviewed a student at their grade level in order to evaluate the student’s thinking with respect to that summer’s content topic.

Seminars on pedagogical practice. During each of the 3 academic years, participants attended 6 half-day seminars. The seminars analyzed practices advocated in the Teaching Standards (NCTM, 1991). Topics included alternative assessment, cooperative groups, classroom discourse, worthwhile mathematical tasks, and writing in mathematics. These practices were discussed in the seminars, and suggestions for their implementation were presented.
Collaboration and reflection. Each seminar had a formal segment where participants shared ideas and successful practices as well as informal opportunities for sharing during breaks and activities. Also, as part of each district’s contribution, participants were given a half-day per semester for collaboration within their building.

Each year participants videotaped and analyzed two classroom lessons. They also kept a reflective journal, and at the end of each year they provided a summary of their journals for project staff. This summary highlighted changes that had occurred in their teaching and identified their goals for the coming year. The final journal summary reflected on the entire 3 years and discussed how their instructional practice had been influenced by the project.”

(b)

Subjects
“The subjects were 49 middle-grade (4-8) teachers participating in Project LINCS, a 3-year professional development project funded by the National Science Foundation. The participants, all volunteers, were drawn from a commuting distance of a midwestern university and received 5 semester hours of graduate credit and a summer stipend for each year of participation. They were divided into two sections for the summer courses. Group 1 comprised the Grades 4-5 teachers and Group 2 the Grades 6-8 teachers….

Intervention Program
Each year of Project LINCS consisted of a 4-week summer session and six half-day seminars during the academic year. Each summer the program emphasized a different subject-matter content, with geometry being the focus area during the second summer. The geometry program consisted of a mathematics content course on geometry, which met 3 hours a day for 4 days a week, and a research seminar, which met for 2 hours once each week. The two sections of the content course were taught by two mathematics education faculty, one of whom was the second author. The research seminar was conducted by the first author. The third author coordinated the academic-year seminars.

The geometry course focused on the exploration of two- and three-dimensional shapes through recognition, analysis, and informal deduction, with greater emphasis on analysis and informal deduction. Instructors adopted an instructional approach that has been described as ‘teaching via problem solving’ (Schroeder & Lester, 1989). Using this approach, each session commenced with the presentation of a problem that embodied key aspects of the topic. Participants worked on the problem in small groups and then shared their solutions in a class discussion. During the class discussion, solution strategies were refined, extension problems were formulated and solved, connections were identified, and discussions ensued on the van Hiele level of key tasks associated with the problem. The textbook for the course, Geometry: An Investigative Approach (O’Daffer & Clemens, 1992), was compatible with the ‘teaching via problem solving’ approach. As part of the course, the participants also developed an instructional unit and accompanying assessment plan for their respective grade levels.

The research seminar presented the van Hiele theory of cognitive development and instruction in geometry. This was followed by an examination of the research on the van Hiele levels of
students (Burger & Shaughnessy, 1986; Fuys et al., 1988; Mayberry, 1983; Senk, 1989). Research on geometry text materials (Fuys et al., 1988; Whitman & Komenaka, 1990) and the results of national and state assessments in geometry were also reviewed. In addition to the research readings, teachers had the option of either interviewing a student at the grade level they teach or analyzing instructional activities in their textbooks by van Hiele levels. For the student interviews, 36 items developed by Mayberry (1981) for assessing the first three van Hiele levels were used. The textbook analysis used the methodology described in Fuys et al. (1988). Both activities were designed to help make the research real to the participants by linking the results and methodology either to one of their own students or to their own textbooks. The interviews also served to give the teachers greater insight into their students’ thinking about geometry.”

The Study
The VCE Mathematics Professional Learning Program

“The VCE (Victorian Certificate of Education) mathematics professional learning program (PLP) for senior secondary mathematics was designed for practising secondary teachers of mathematics who had no experience of teaching advanced senior secondary mathematics and who had not completed the recommended qualifications. The teachers wanted a program that was situated and practice based and that would enable them to learn the mathematics of grades 11 and 12 (VCE Mathematical Methods and Further Mathematics) and the methods of teaching this mathematics. In this professional learning program, teachers were students of mathematics (Leiken & Winicki Landman, 2001), and this mathematics included algebra, functions and graphs, rates of change and calculus, probability and data analysis, including statistical modelling.

The PLP involved seminars as well as self-directed practice-based inquiry and portfolio development. Teachers attended 21 three-hour seminars conducted fortnightly on an afternoon during school terms over a school year. We provided each of the teachers with a CAS (computer algebra system) calculator and at least two different textbooks for each of the VCE mathematics subjects, and we encouraged the teachers to purchase a mathematics dictionary of their choice.

We used both mathematics and professional learning tasks during the seminars. A typical session began with finding out what the teachers knew about the mathematics concepts and procedures concerning a mathematics task (e.g., Figure 1). One of us would scaffold discussion of conceptual or procedural ideas relevant to the task. Teachers would then work on mathematics tasks, including closed tasks such as the one in Figure 1, open-ended tasks, tasks designed to explore concepts, tasks with the purpose of deriving or proving rules, or mathematics-related professional learning tasks. These professional learning tasks (Silver et. al., 2007) included analysis of mathematics problems and activities, analysis of teaching materials and teaching strategies, design of mathematics problems and activities, analysis of students’ solutions, and review of VCE subject examiners’ reports. The session would end with a discussion to summarise the key mathematical concepts and procedures. Teachers reflected on their experience of working on these tasks and the outcomes of this discussion were used to identify potential student misconceptions and key points for teaching.

![Figure 1. A task on transformation of trigonometric functions.](image)
The sequence of topics for the seminars followed the sequence normally used by teachers of the grade 12 subjects. Hence we included seminars with discussions about the formal assessment tasks of the VCE subjects at roughly the same time that teachers were designing and assessing students with these tasks. Experienced senior secondary mathematics teachers also conducted a few sessions in the program. Their sessions focused on curriculum knowledge, long-term planning for teaching and assessment, resources for teaching, and tasks used for teaching and assessment.

The practice-based component of the program occurred in the teacher’s school between seminars. We encouraged participants to establish a mentor relationship with an experienced teacher of senior secondary mathematics to support their school-based self-directed inquiry. We recommended that they negotiate with their colleagues to observe and/or team-teach grade 11 or 12 mathematics lessons, observe students doing mathematics (in lessons or by tutoring students), reflect on observations, analyse student work, research and critique teaching and assessment resources and materials, and to participate in the moderation processes of student assessment for these subjects. The teachers documented their learning in an annotated portfolio that they presented to their peers in the final session of the program.”