Interventions that Engaged Teachers with Student Instructional Materials in Mathematics


The Summer Institute

“During the summer of 1999, 22 middle and secondary teachers participated in the integrated science and mathematics institute. Of these 22, 10 had previously participated in an integrated science and mathematics program at WSU. The teacher participants had diverse backgrounds ranging from 2 to 35 years of teaching experience and from no professional development experience to master’s degrees. Additionally, the areas in which the teachers specialized were quite different: four special education teachers, four mathematics and four science high school teachers, and 11 middle school teachers. The three districts involved were a large urban district, a suburban district, and a smaller urban and rural mixture. Student populations ranged from 75% African American students to 85% Caucasian students. These districts were chosen because they are local to the university, and WSU’s preservice teachers are often placed with these districts for field experiences.

The summer institute was intensive with 72 contact hours of class over a 4-week time period, meeting 8 hours a day for 3 days a week. We immersed the teachers in inquiry-based learning environments, in which they worked on integrated science and mathematics units in cooperative groups of three or four. The general structure of the institute involved teachers spending two thirds of each day working on content units and the remainder of the day considering pedagogical issues and developing such units for use in their own classrooms.

Two different cooperative groupings were used. For the science and mathematics investigations, teachers were grouped heterogeneously with the requirements that the teachers in the group could not all teach the same grade level, or be in the same district, or teach the same subjects. For the development of units, cooperative groups were formed by same, or similar, grade-level teachers, since these teachers shared similar curricula and were often from the same district. Including both elementary and secondary science and mathematics teachers within the same groups for content investigations effectively expanded the resources and expertise available to groups in both content and pedagogical knowledge. Rich discussions resulted from these heterogeneous groups, often involving topics of vertical curriculum alignment and effective pedagogical strategies. Heterogeneous grouping typically developed mutual respect and cooperation among the different grade level and topic teachers.

To best model standards-based integrated science and mathematics teaching practices, we team-taught the institutes. In this way, teachers experienced teaching from both the science and mathematics perspectives and gained pedagogical knowledge of both disciplines. Master’s degree program students who were also secondary science and mathematics teachers helped facilitate the institutes. These ‘resource’ teachers provided real classroom connections that aided participants in transferring the institute experiences to the precollege classroom.

Due to the diverse backgrounds, teaching assignments, and teaching environments of the teachers, the content of the institute was matched to grade 4-12 strands of the science and mathematics standards, with topics chosen for their importance and integration aspects. Content investigations started with the most fundamental concepts, usually encountered in the earlier grades, and built up to
the concepts and applications of the upper grades. Even though the content was consistent with grades 4-12 standards, the teacher participants analyzed the content at an adult level in order to develop the conceptual understanding necessary to teach effectively.

We used a combination of commercial curricula and curricula we designed. Commercial resources included *Mathematical Modeling in Our World* (The Consortium for Mathematics and Its Applications, 1998a) and physics education materials (Arons, 1997; McDermott, 1996). The integrated science and mathematics units we designed ourselves were adapted from preservice teacher course activities (Basista, 1998a, 1998b). When designing the units, we took great care to maintain conceptual development for both disciplines. Indeed, we chose many of the specific science and mathematics topics not only for their importance in the teachers’ curricula, but also because the topics lent themselves to a high degree of integration. In every case, we made no assumptions about the backgrounds of the teachers. Each unit started with the most fundamental concepts and built teacher understanding from that basis. Since 1997, we have utilized units such as motion and graphing; shadows and proportional reasoning; and simple machines and proportional reasoning. Refer to Table 1 for the topics covered in 1999.

The integrated science and mathematics units were of a guided discovery format, with facilitator checkpoints included after conceptually connected sections. At the checkpoints, we utilized questioning techniques not only to deepen the teachers’ understanding, but also to model effective questioning strategies. At these checkpoints, we often discussed pedagogical issues related to teaching the material in grades 4-12 classrooms. We assigned daily homework over the sections completed to help solidify the teachers’ understandings of the content and to provide further examples of applications of the concepts.

The pedagogical issues addressed during the institutes related directly to the standards, their implementation, and assessment. These topics included comparisons between inquiry and traditional environments, assessing students’ prior understandings, methods of modifying and/or developing inquiry-based activities, cooperative learning techniques, development of in-depth conceptual understanding, development of problem-solving skills, integration of science and mathematics, reflection on one’s teaching, and authentic assessment techniques. For a sample of pedagogical content covered in the summer institute, see Table 1.

About halfway through the institute, the class was divided into groups of teachers who taught similar grade levels so that they could develop integrated science and mathematics units for use in their classrooms. At this point, the teachers began to apply the science, mathematics, and pedagogical content knowledge they had acquired during the institute to their own classrooms. During the final two days of the institute, the teachers team-taught lessons from their developed units for the class and received peer and instructor feedback.

**Academic Year Support Activities**

We visited the teachers’ classrooms three times during the academic year to observe them, to model teaching methods, and to provide feedback about their teaching practices. During the academic year, the teachers attended three workshops, in which they shared the results of their efforts. During the workshops, pedagogical issues and district issues were frequently discussed. We encouraged teachers to maintain contact with us through phone and email.
Throughout the academic year, the teachers built portfolios documenting their efforts in modifying their teaching practices. These portfolios included lessons they had taught in their classroom, together with reflections, student feedback, and results. Teachers documented their efforts in implementing inquiry and cooperative teaching practices, developing their students' in-depth content understanding and problem-solving skills, and utilizing forms of authentic assessment.”

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<tr>
<th>Table 1</th>
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<td><strong>Science Content</strong></td>
<td><strong>Mathematical Content</strong></td>
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<td>Measuring heights and distances</td>
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<td>Levers</td>
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Details of the Mathematics Community Continuum

“Our community continuum was a long-term, on-site professional development project that existed among a school of education, a mathematics department, and a high need, urban mini-district (charter school). The members of this community were drawn from five primary groups: prospective elementary education teachers (interns), practicing elementary teachers, practicing high school teachers (teacher leaders), school administrators, and university faculty/researchers (Figure 1). Based on the current research recommendations outlined in the previous section, we (researchers/instructors) created this effort in response to this school's history of low student achievement and our Nation's promise of a Highly Qualified teacher in every classroom, particularly in mathematics.

This school provided a unique and rich context for this project for the following primary reasons: a large percentage of under-qualified teachers, flexibility to create and implement curriculum innovation, and an urban setting to explore the design and implementation of mathematics professional development. Additionally, having worked with teachers in this charter school's secondary location, we had a working relationship with staff and administrators, and were invited into the community to develop and facilitate professional development at the elementary level (Benken, Brown, & Smith, 2007). Previously developed trust with this charter school system helped to facilitate generation and commencement of the community, as all members were comfortable with our presence in both the building and their classrooms, thereby circumventing issues of power that often prevent professional development initiatives from achieving maximum success (e.g., Bier, Kazemi, Horn, Stevens, & Peck, 2007). Finally, administration at this site found the multi-level mentoring, which included prospective teachers, to be an exciting context for meeting the daily classroom needs and long-term professional growth of their teachers.

Key components of this professional development experience included:

- Collaboration among all constituents surrounding a focus on students' improved conceptually based understandings and achievement, as well as increased performance on high-stakes state assessments;
- Attention to on-going intellectual and emotional support of the community through conversations surrounding practical needs of practice, generation of a trusting working relationship, and new learning that was occurring during sessions and classroom activities;
- Bi-monthly professional development sessions that worked with teachers on their understanding of general pedagogy, mathematical content (and other subject-matter content) and pedagogical content knowledge, and how to implement reform curricula in their classrooms; these sessions were held during the teachers' site-established professional meeting time;
- An approach to professional development instruction that allowed teachers to work on their specific classroom needs; in this way session instruction was differentiated to meet the needs of the individual learners (i.e., teachers);
- Mentoring partnerships that involved interns and practicing teachers working together in the same classroom;
• Summer workshops that focused on the interaction among constituents to facilitate school's articulated goals; and
• On-going, guided reflection with teachers, teacher leaders, interns, and building administrators through journals (prompt based and open-ended) and email correspondence.”
“We begin with discussion of ways in which graphing technology has introduced a greater focus on relationships between the x’s and y’s of algebra and graphs of functions on the Cartesian coordinate system, a shift from algebraic to analytic perspectives (as described in greater detail in Chazan, 2000, pp. 70–75)…”

1. Background: graphing technology and school algebra
As suggested earlier, over the past decade, school mathematics has moved to assimilate graphing technology into practices of teaching and learning. For example, in the United States, the National Council of Teachers of Mathematics (NCTM,1989, 2000) recommends the use of graphing technology in order to support learning mathematics with understanding. And, as highlighted in a recent ICMI study, such recommendations are not limited to the United States; ‘many educational jurisdictions make explicit recommendations on the use of calculators and computers for algebra teaching,’ for either teacher demonstration or student problem solving (Stacey & Kendal, 2004, p. 342)…”

2. Teachers and changes to school algebra
The availability of technology and of curricula taking new approaches to school algebra have spawned studies of teacher thinking and knowledge and behavior in school algebra. Many of these studies have focused particularly on pre-service and in-service teachers’ thinking about functions (e.g., Cooney & Wilson, 1993; Even, 1993; Haimes, 1996; Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Lloyd & Wilson, 1998; Sánchez & Llinares, 2003; Wilson, 1994). Based on studies of student difficulties with the notion of function, many of these studies focus on whether teachers have the sorts of mathematical knowledge, particularly of functions, that will support the implementation of function-based approaches to school algebra. Other studies that explore teachers undergoing processes of curricular change in school algebra consider the structure or input of educational programs that support teachers’ attempts to reform-oriented curriculum (e.g., Cooney, 1994), teachers’ beliefs and attitudes towards different types of curriculum (e.g., Heid, Blume, Zbiek, Edwards, & Barbara, 1999), development of teachers’ sensitivity to, and knowledge of, students in the context of reform-oriented curriculum (e.g., Lloyd & Wilson,1998), and teachers’ conceptions of cooperation and exploration components of a reform-oriented curriculum and classroom culture (e.g., Lloyd, 1999).”
“This study examined three middle school mathematics teachers with varying levels of experience that were teaching in suburban schools. The three teachers involved in the study were enrolled in a master’s degree program in Atlanta, Georgia with one of the researchers serving as the instructor. The teachers volunteered to take part in this study with the understanding that they would consider meaningful forms of mathematics instruction, specifically model eliciting problems, to supplement their mathematics curriculum. They agreed to identify underlying concepts and skills of each model eliciting problem to begin mapping their curriculum for one year. Moreover, they agreed to focus their attention on their students’ mathematical thinking to better understand the nature of mathematics and their students’ perceptions of the skills and concepts in particular problems…

Teachers met weekly with researchers in workshop environments where they were presented with one model eliciting problem every other week for 14 weeks. They collaboratively grappled with solving the problems as well as identifying and visually mapping key concepts, skills and important mathematical ideas that were embedded within the rich problem. They used national, state and school standards to further document the types of concepts represented in each problem. They went on to categorize smaller problems sets and sets of symbolically represented problem that were aligned with the model eliciting problems that could serve as follow up problems or homework sets. Again these smaller problems were combined within the mapping as well. After sharing their own ideas and representations, they agreed to use these problems in their own classrooms.

During classroom implementation with researchers present, teachers were encouraged to recognize and analyze student interpretations as they were continually revised and refined. This in turn aided their understanding of the skills and concepts embedded within the model eliciting problem. Independently they would reflect, revise and refine their own thinking about the mapping of the concepts and skills found within the model eliciting problems. They would bring their thoughts and ideas back to share with their colleagues in subsequent workshop sessions. Studying each other’s mappings as a group afforded the opportunity to both consider the development of their ideas, to discuss students’ thinking in regards to particular model eliciting problems as well as discuss the pedagogical implications of using model eliciting problems in their classrooms.”

“While curriculum materials cannot alone provide teachers the knowledge needed to teach in new ways, they can provide leverage for changes in practice and hence learning. A particular question is what kinds of learning and new knowledge about content and children's thinking can an innovative curriculum program motivate? In particular, what basis for making new knowledge claims about mathematics and about students might the teaching of such a curriculum provide teachers?...

Ball and Cohen (1996) have proposed that properly designed textbooks have the potential to help scale change. Research on children's thinking about multidigit operations provides a backdrop against which to pose questions about the nature and extent of what teachers might learn as they teach using such materials. To consider this question, examples have been culled from a set of interviews with teachers about what they learned implementing Investigations in Number, Data, and Space (1995-1998, TERC). Participants consisted of all the teachers in grades three, four and five at a single elementary school, located in a district that has adopted Investigations as its elementary mathematics program.”

“First, reform-oriented curricula take a novel approach to content by emphasizing reasoning, problem solving, and modeling where students are encouraged to make sense of the mathematics they are learning and to use procedures that they understand. This approach likely contributes to teacher learning as well as student performance. While evidence about the positive impact of reform curricula on students’ learning of mathematics has been extensively documented (Senk & Thompson, 2003), studies that focus on the impact of those curricula in supporting teachers learning are much needed. Second, reform-oriented curricula offer extensive information for teachers such things as importance of particular content, different ways of representing a mathematical topic, various strategies students may use and why they work, relationships to other topics, and sample dialogue a teacher may have with students on a particular mathematical idea.”
“A brief summary of the seminars over the 2-year period is provided so that the readers will have some understanding of the mathematical content to which the teachers were exposed and how they responded to the content.

An introductory meeting in the spring of 1992 was intended to acquaint the teachers with one another and to give them an indication of what to expect from the seminars, and to give us an indication of what to expect from the teachers. During the session, the teachers were given an excerpt from a lesson on fractions (Borko et al., 1992). In the ensuing discussion, one teacher said:

> Maybe I’m way off the wall, but I don’t teach kids to flip numbers upside down.... So we review multiplying fractions....Then I put up a problem with division. (The teacher wrote a division problem on the board and drew two large Xs through the fractions while reciting the following.) I say, ‘Follow these lines and multiply, and you got your answer. Just go ‘I hate math; I hate math. Boy! Do I really hate math!’ (See chapter 8 for more detail.)

The other teachers responded positively to this method. They felt that teaching fractions was extremely difficult; any ‘gimmicks’ would be useful. They indicated that they did not think it was possible to ‘teach fractions with understanding,’ and some also used Explorer calculators for multiplying and dividing fractions. They felt that students’ demands for the answers prevented them from teaching conceptually.

**Year 1.** In the fall of 1992 we held 2 full-day seminars for the teachers. We began by discussing data on how children compare decimal numbers (from Resnick et al., 1989), then worked on place value with decimal numbers via the Blocks Microworld (P. Thompson, 1992). In the afternoon of the first day the teachers completed the Content Knowledge Assessment instrument. We made copies of the completed tests and discussed them among ourselves in terms of the areas on which we should focus our efforts. The tests were returned to the teachers unmarked, and the second day was devoted to discussing the items on that test. The teachers became very involved in considering their own responses and those of the others, then thinking about how their students would react to some of the items. Some of the items had been used with students (Armstrong & Larson, 1995), and the ways in which students thought about those items and solved them were discussed with the teachers. (When relevant, each teacher’s work on this assessment is discussed in the individual case studies.)

For the remainder of the year, approximately half the seminars were presentation-focused—that is, a researcher prepared a presentation based on research with children. The presentations were informal, and there were questions and discussions throughout the presentations. When the presentations were made by visiting researchers from outside the university, a few additional teachers were invited, so that the audience was approximately a dozen. (The presentation-based seminars were substantially the same as the written versions of the presentations appearing in *Providing a Foundation for Teaching Mathematics in the Middle Grades* [J. Sowder &
The first two seminars were intended to lead the teachers to see the value of sense-making as part of the enterprise of teaching. A presentation on rational number sense led to practice with mental computation and estimation and to examination of sense-making with operations and algorithms.

The next four seminars focused on developing the teacher’s understanding of fractions and fraction operations. Presentations by Mack, Armstrong, Bezuk, and Kieren provided the teachers with research-based ways of presenting critical ideas about fractions and fraction operations. Examples of students’ thinking and working with fractions challenged the teachers to think about their role in teaching fractions in meaningful ways. The fourth seminar was devoted to discussion of the results of the teachers’ students’ work on the Fraction Understanding Test (provided in Appendix F). The items tested for conceptual understanding rather than algorithmic skill. The teachers were surprised and distressed with the results. Although they recognized that they were not responsible for the poor performance (the tests were administered after students had been in their classes for less than 2 months), they also realized that until this seminar they had little comprehension of what their students knew and did not know, thus making it difficult for them to base instruction on students’ knowledge. (This seminar was summarized and analyzed in Armstrong, Philipp, & J. Sowder, 1993.)

A more holistic look at both whole number and rational number operations was the subject of the presentation ‘Addressing the Story-Problem Problem’ by L. Sowder. He discussed the connections between the operations and the real-world applications, focusing on what elements in a situation lead to choosing the correct operation.

The next three seminars were informal; they focused on critical incidents in the teachers’ own classrooms and on discussion of the previous presentations. The teachers compared ways that their own planning for instruction on fractions was changing.

In the two following seminars we turned to the topic of proportional reasoning; the discussion was based on a presentation by Lamon. Proportional reasoning as multiplicative reasoning was discussed in some detail. These seminars led into Harel’s presentation in which he outlined students’ progress through additive reasoning into multiplicative reasoning. The final presentation of the year, by P. Thompson, focused on quantitative reasoning in both simple and complex situations.

For the closing seminar of the first year we chose several transcript excerpts from the seminars during which teachers had struggled with mathematical concepts and had finally came to a deep understanding of them. The teachers were given the assignment of reading excerpts and providing written reactions to them at a later date. To set the stage for this assignment, the
investigators each earlier wrote reflections on the year’s work and shared them with the teachers at this seminar. For the remainder of the seminar, the teachers talked informally about what they had learned and how they had changed over the course of the year. The conversations focused on the seminars, our classroom observations, their own planning and insights, and their classroom interactions with students.

**Year 2.** During Year 2, several of the topics introduced during the Year-1 seminars were revisited, sometimes through discussions of (sometimes videotaped) segments of the participating teachers’ classroom rational number lessons that had been observed by the researchers and sometimes through revising the papers written by the presenters of Year 1. The first seminar of the year was devoted to eliciting individual teacher reflections, partly to determine ways to provide seminars of most benefit to the teachers at this stage in their participation in the project. Teachers spoke about their mathematical goals for the year, their mathematical expectations for their students for the year, their roles as teachers, perceived obstacles in teaching mathematics, the growth of students from additive to multiplicative reasoning, changes they were making or would like to make in their mathematics teaching, and what each hoped to gain from the project during the coming year.

In seminar discussions of observed teachers’ classroom lessons (sometimes with videotaped segments presented), the importance of consistently relating the part to the unit was an issue in both the fractions and decimal lessons being discussed; all of the teachers recognized this as an issue in their own classrooms. Issues related to the use of models for rational numbers also arose in these seminars. Before one seminar, two researchers had visited the same teacher a few days apart; the second had the opportunity to see implemented the first’s suggestion to incorporate proportional reasoning into a lesson. Describing this lesson sparked a discussion of teachable moments—awareness of situations in which opportunities to develop important ideas, in this case proportional reasoning, arise.

The one topic tested on the initial Content Knowledge Test but not addressed during Year 1 was that of weighted average in rate problems. This difficult topic was approached in Year 2 through the use of P. Thompson’s *Over and Back* (1994) microworld.

Just as in Year 1, most of one seminar was devoted to discussion of students’ fraction-understanding-test and interview results. The teachers appreciated the limitations of the pencil-and-paper instrument, even though it focused on conceptual learning and the greater richness of the responses in interviews in which answers could be probed for reasoning and in which misinterpretations of the problems were evident.

During these Year-2 seminars, even more than in the Year-1 meetings, the teachers often raised questions or shared classroom experiences that led to extended discussions (e.g., Darota gave students a problem to do individually so that she could work on report cards, but the problem instead turned into an extended lesson on ratio). Issues about standardized tests and textbooks were raised repeatedly. The importance of the teachers’ having deep understanding of the content, the big ideas within a topic, the connections among topics—instead of merely presenting interesting problems that are not necessarily part of a bigger, overall picture of rational
numbers—was recognized by the teachers and was raised by them more than once during the Year-2 seminars.

Year 2 concluded with a seminar in which we, the researchers, explained that we would now be trying to tell what had been learned from this project, and the teachers were asked to reflect on their participation and to tell what had been learned from their points of view. The teachers spoke quite passionately about how much they had learned and about the need for all teachers to have more opportunities to focus on mathematics during professional development.”


(a)

**Participants in the Intervention**

“The subjects were 48 middle-grades (4-8) teachers participating in Project LINCS. These participants, all volunteers, came from 30 schools and 12 school districts within a 50-mile radius of a Midwestern university. Pairs of participants from the same school were actively sought, but only 32 came from schools with 2 or more participants. Their mean number of years of teaching experience was 13.6….

**Intervention Program**

Project LINCS was a 3-year intervention program designed to enhance teachers’ knowledge through annual 4-week summer content courses, accompanying 8-hour research seminars on student cognition, and 6 half-day seminars during the academic year focusing on pedagogical practice. The intervention also incorporated structured on-going teacher collaboration and reflection.

**Content courses.** The summer content courses addressed probability and statistics in year 1, geometry in year 2, and algebra in year 3. The probability and statistics course emphasized the exploration of data and the use of simulation to determine probabilities. Visual displays and descriptive statistics were used to examine characteristics and patterns in data; and theoretical probabilities, simulations, and data analysis were used to solve a wide variety of probability problems. The geometry course focused on the exploration of two- and three-dimensional shapes using the van Hiele (1959/1985) levels of recognition, analysis, and informal deduction as a basis for instruction. In particular, the course incorporated an investigation of polygons and their properties; tessellations; polyhedra and their properties; length, area, and volume measures; and motion geometry. The algebra course explored families of functions in problem contexts. This exploration used graphical, tabular, and symbolic representations to investigate linear, quadratic, and exponential functions. The instructional approach adopted in each course can be described as ‘teaching via problem solving’ (Schroeder & Lester, 1989) and modeled the pedagogy advocated in the half-day seminars. Computers and graphics calculators were used in all courses.

**Research seminars on student cognition.** The companion research seminar reviewed and discussed research findings on students’ cognition in each of the three content areas and reflected on the implications of these for classroom instruction. The seminars examined the research on the development of probabilistic thinking (Shaughnessy, 1992); van Hiele levels (Fuys, Geddes. & Tischler, 1988); and students’ understanding of variables and their uses (Kieran & Chalouth, 1993). Each year, participants also interviewed a student at their grade level in order to evaluate the student’s thinking with respect to that summer’s content topic.

**Seminars on pedagogical practice.** During each of the 3 academic years, participants attended 6 half-day seminars. The seminars analyzed practices advocated in the *Teaching Standards* (NCTM, 1991). Topics included alternative assessment, cooperative groups, classroom discourse, worthwhile mathematical tasks, and writing in mathematics. These practices were discussed in the seminars, and suggestions for their implementation were presented.
Collaboration and reflection. Each seminar had a formal segment where participants shared ideas and successful practices as well as informal opportunities for sharing during breaks and activities. Also, as part of each district’s contribution, participants were given a half-day per semester for collaboration within their building.

Each year participants videotaped and analyzed two classroom lessons. They also kept a reflective journal, and at the end of each year they provided a summary of their journals for project staff. This summary highlighted changes that had occurred in their teaching and identified their goals for the coming year. The final journal summary reflected on the entire 3 years and discussed how their instructional practice had been influenced by the project.”

(b)

Subjects
“The subjects were 49 middle-grade (4-8) teachers participating in Project LINCS, a 3-year professional development project funded by the National Science Foundation. The participants, all volunteers, were drawn from a commuting distance of a midwestern university and received 5 semester hours of graduate credit and a summer stipend for each year of participation. They were divided into two sections for the summer courses. Group 1 comprised the Grades 4-5 teachers and Group 2 the Grades 6-8 teachers….

Intervention Program
Each year of Project LINCS consisted of a 4-week summer session and six half-day seminars during the academic year. Each summer the program emphasized a different subject-matter content, with geometry being the focus area during the second summer. The geometry program consisted of a mathematics content course on geometry, which met 3 hours a day for 4 days a week, and a research seminar, which met for 2 hours once each week. The two sections of the content course were taught by two mathematics education faculty, one of whom was the second author. The research seminar was conducted by the first author. The third author coordinated the academic-year seminars.

The geometry course focused on the exploration of two- and three-dimensional shapes through recognition, analysis, and informal deduction, with greater emphasis on analysis and informal deduction. Instructors adopted an instructional approach that has been described as ‘teaching via problem solving’ (Schroeder & Lester, 1989). Using this approach, each session commenced with the presentation of a problem that embodied key aspects of the topic. Participants worked on the problem in small groups and then shared their solutions in a class discussion. During the class discussion, solution strategies were refined, extension problems were formulated and solved, connections were identified, and discussions ensued on the van Hiele level of key tasks associated with the problem. The textbook for the course, Geometry: An Investigative Approach (O’Daffer & Clemens, 1992), was compatible with the ‘teaching via problem solving’ approach. As part of the course, the participants also developed an instructional unit and accompanying assessment plan for their respective grade levels.

The research seminar presented the van Hiele theory of cognitive development and instruction in geometry. This was followed by an examination of the research on the van Hiele levels of
students (Burger & Shaughnessy, 1986; Fuys et al., 1988; Mayberry, 1983; Senk, 1989). Research on geometry text materials (Fuys et al., 1988; Whitman & Komenaka, 1990) and the results of national and state assessments in geometry were also reviewed. In addition to the research readings, teachers had the option of either interviewing a student at the grade level they teach or analyzing instructional activities in their textbooks by van Hiele levels. For the student interviews, 36 items developed by Mayberry (1981) for assessing the first three van Hiele levels were used. The textbook analysis used the methodology described in Fuys et al. (1988). Both activities were designed to help make the research real to the participants by linking the results and methodology either to one of their own students or to their own textbooks. The interviews also served to give the teachers greater insight into their students’ thinking about geometry.”