Interventions that Engaged Teachers in Considering Student Thinking about Mathematics

Excerpted from Basista, B. & Mathews, S. (2002). Integrated science and mathematics professional development programs. *School Science and Mathematics, 102* (7), 359–70.

The Summer Institute

"During the summer of 1999, 22 middle and secondary teachers participated in the integrated science and mathematics institute. Of these 22, 10 had previously participated in an integrated science and mathematics program at WSU. The teacher participants had diverse backgrounds ranging from 2 to 35 years of teaching experience and from no professional development experience to master's degrees. Additionally, the areas in which the teachers specialized were quite different: four special education teachers, four mathematics and four science high school teachers, and 11 middle school teachers. The three districts involved were a large urban district, a suburban district, and a smaller urban and rural mixture. Student populations ranged from 75% African American students to 85% Caucasian students. These districts were chosen because they are local to the university, and WSU's preservice teachers are often placed with these districts for field experiences.

The summer institute was intensive with 72 contact hours of class over a 4-week time period, meeting 8 hours a day for 3 days a week. We immersed the teachers in inquiry-based learning environments, in which they worked on integrated science and mathematics units in cooperative groups of three or four. The general structure of the institute involved teachers spending two thirds of each day working on content units and the remainder of the day considering pedagogical issues and developing such units for use in their own classrooms.

Two different cooperative groupings were used. For the science and mathematics investigations, teachers were grouped heterogeneously with the requirements that the teachers in the group could not all teach the same grade level, or be in the same district, or teach the same subjects. For the development of units, cooperative groups were formed by same, or similar, grade-level teachers, since these teachers shared similar curricula and were often from the same district. Including both elementary and secondary science and mathematics teachers within the same groups for content investigations effectively expanded the resources and expertise available to groups in both content and pedagogical knowledge. Rich discussions resulted from these heterogeneous groups, often involving topics of vertical curriculum alignment and effective pedagogical strategies. Heterogeneous grouping typically developed mutual respect and cooperation among the different grade level and topic teachers.

To best model standards-based integrated science and mathematics teaching practices, we team-taught the institutes. In this way, teachers experienced teaching from both the science and mathematics perspectives and gained pedagogical knowledge of both disciplines. Master's degree program students who were also secondary science and mathematics teachers helped facilitate the institutes. These 'resource' teachers provided real classroom connections that aided participants in transferring the institute experiences to the precollege classroom.

Due to the diverse backgrounds, teaching assignments, and teaching environments of the teachers, the content of the institute was matched to grade 4-12 strands of the science and mathematics standards, with topics chosen for their importance and integration aspects. Content investigations started with the most fundamental concepts, usually encountered in the earlier grades, and built up to the concepts and applications of the upper grades. Even though the content was consistent with grades 4-12 standards, the teacher participants analyzed the content at an adult level in order to develop the conceptual understanding necessary to teach effectively.

We used a combination of commercial curricula and curricula we designed. Commercial resources included *Mathematical Modeling in Our World* (The Consortium for Mathematics and Its Applications, 1998a) and physics education materials (Arons, 1997; McDermott, l996). The integrated science and mathematics units we designed ourselves were adapted from preservice teacher course activities (Basista, 1998a, 1998b). When designing the units, we took great care to maintain conceptual development for both disciplines. Indeed, we chose many of the specific science and mathematics topics not only for their importance in the teachers' curricula, but also because the topics lent themselves to a high degree of integration. In every case, we made no assumptions about the backgrounds of the teachers. Each unit started with the most fundamental concepts and built teacher understanding from that basis. Since 1997, we have utilized units such as motion and graphing; shadows and proportional reasoning; and simple machines and proportional reasoning. Refer to Table 1 for the topics covered in 1999.

The integrated science and mathematics units were of a guided discovery format, with facilitator checkpoints included after conceptually connected sections. At the checkpoints, we utilized questioning techniques not only to deepen the teachers' understanding, but also to model effective questioning strategies. At these checkpoints, we often discussed pedagogical issues related to teaching the material in grades 4-1 2 classrooms. We assigned daily homework over the sections completed to help solidify the teachers' understandings of the content and to provide further examples of applications of the concepts.

The pedagogical issues addressed during the institutes related directly to the standards, their implementation, and assessment. These topics included comparisons between inquiry and traditional environments, assessing students' prior understandings, methods of modifying and/or developing inquiry-based activities, cooperative learning techniques, development of in-depth conceptual understanding, development of problem- solving skills, integration of science and mathematics, reflection on one's teaching, and authentic assessment techniques. For a sample of pedagogical content covered in the summer institute, see Table 1.

About halfway through the institute, the class was divided into groups of teachers who taught similar grade levels so that they could develop integrated science and mathematics units for use in their classrooms. At this point, the teachers began to apply the science, mathematics, and pedagogical content knowledge they had acquired during the institute to their own classrooms. During the final two days of the institute, the teachers team-taught lessons from their developed units for the class and received peer and instructor feedback.

Academic Year Support Activities

We visited the teachers' classrooms three times during the academic year to observe them, to model teaching methods, and to provide feedback about their teaching practices. During the academic year, the teachers attended three workshops, in which they shared the results of their efforts. During the workshops, pedagogical issues and district issues were frequently discussed. We encouraged teachers to maintain contact with us through phone and email.

Throughout the academic year, the teachers built portfolios documenting their efforts in modifying their teaching practices. These portfolios included lessons they had taught in their classroom, together with reflections, student feedback, and results. Teachers documented their efforts in implementing inquiry and cooperative teaching practices, developing their students' in-depth content understanding and problem-solving skills, and utilizing forms of authentic assessment."

Table 1
Summer 1999 Topics

Science Content	Mathematical Content	Pedagogical Content
Shadows	Proportional reasoning	Science and mathematics standards
Measuring heights and distances	Geometry	Inquiry
Levers	Multiple representations (graphs, diagrams, symbols)	Integration of science with mathematics
Hooke's Law	Logistics modeling	Developing problem solving skills
Population growth	Modeling vs. problem-solving	Authentic/alternative assessment
Population growth	Exponential growth	Cooperative learning
		Modifying and developing inquiry lessons
		Questioning techniques
		Reflective teaching practices
		Facilitating inquiry lessons

Excerpted from Clark, K. K. & Schorr, R. Y. (2000). Teachers' evolving models of the underlying concepts of rational number. ***Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 22*****.**

"This study examined three middle school mathematics teachers with varying levels of experience that were teaching in suburban schools. The three teachers involved in the study were enrolled in a master's degree program in Atlanta, Georgia with one of the researchers serving as the instructor. The teachers volunteered to take part in this study with the understanding that they would consider meaningful forms of mathematics instruction, specifically model eliciting problems, to supplement their mathematics curriculum. They agreed to identify underlying concepts and skills of each model eliciting problem to begin mapping their curriculum for one year. Moreover, they agreed to focus their attention on their students' mathematical thinking to better understand the nature of mathematics and their students' perceptions of the skills and concepts in particular problems…

Teachers met weekly with researchers in workshop environments where they were presented with one model eliciting problem every other week for 14 weeks. They collaboratively grappled with solving the problems as well as identifying and visually mapping key concepts, skills and important mathematical ideas that were embedded within the rich problem. They used national, state and school standards to further document the types of concepts represented in each problem. They went on to categorize smaller problems sets and sets of symbolically represented problem that were aligned with the model eliciting problems that could serve as follow up problems or homework sets. Again these smaller problems were combined within the mapping as well. After sharing their own ideas and representations, they agreed to use these problems in their own classrooms.

During classroom implementation with researchers present, teachers were encouraged to recognize and analyze student interpretations as they were continually revised and refined. This in turn aided their understanding of the skills and concepts embedded within the model eliciting problem. Independently they would reflect, revise and refine their own thinking about the mapping of the concepts and skills found within the model eliciting problems. They would bring their thoughts and ideas back to share with their colleagues in subsequent workshop sessions. Studying each other's mappings as a group afforded the opportunity to both consider the development of their ideas, to discuss students' thinking in regards to particular model eliciting problems as well as discuss the pedagogical implications of using model eliciting problems in their classrooms."

Excerpted from Dole, S., Clark, D., Wright, T., Hilton, G., & Roche, A. (2008). ***Eliciting growth in teachers' proportional reasoning: Measuring the impact of a professional development program.*** **Paper presented at the Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane, Queensland, Australia.**

"A targeted program of professional learning is being incorporated within the teacher meetings, in which new ideas and ways of teaching to connect essential learnings in mathematics and science are shared with teachers….

At the end of the first year of the project, and after the teachers had attended 5 full professional development days and 4 afternoon workshops throughout the year, the teachers completed the BTS a second time."

Excerpted from Ellington, A. J., Whitenack, J. W., Inge, V., Murray, M., & Schneider, P. (2009). ***Assessing K-5 teacher leaders' mathematical understanding: What have the test makers and the test takers learned?***

"We used the Developing Mathematical Ideas [DMI] materials to train and support K-5 Mathematics Specialists…

The DMI materials provided numerous, rich activities and assignments as well as suggestions for ways to assess teachers' progress qualitatively…

The DMI materials (Schifter, D., Bastable, V., & Russell, S., 1999a, 1999b) have been useful instructional materials for addressing issues of teaching and learning mathematics. The central feature of the DMI materials is a series of case studies authored by teachers about how children learn and reason about mathematical concepts. The cases contain a range of examples of children's work as well as interactions among children and/or the teacher. Often, the author of a case provides the reader with a firsthand account of her thought processes as she makes decisions about instructional strategies or conducts individual or whole class discussions. As teachers use these materials, they have opportunities to engage in discussions and explorations about mathematics and mathematics pedagogy.

Because these materials focus extensively on children's reasoning and explorations of fundamental ideas the underpin children's reasoning, these materials might also be said to address mathematical knowledge for teaching [MKT] and pedagogical content knowledge [PCK] for elementary school teachers….

The Numbers and Operations Course
The mathematical content covered in *Numbers and Operations* consists of a detailed study of the base ten system and the four basic operations (addition, subtraction, multiplication, and division). The operations are analyzed in a variety of mathematical contexts including whole and rational number situations. In Summer 2006, the course was offered at three sites across the state for teachers who were beginning the training program to become Mathematics Specialists. In each location, participants met for six hours a day for ten days. The instructional team consisted of university mathematics educators and K-5 teachers who were exemplary participants in prior courses offered through the Mathematics Specialist program. Two books in the DMI series, *Building Systems of Tens* (Schifter, Bastable, & Russell, 1999a) and *Making Meaning for Operations* (Schifter, Bastable, & Russell, 1999b) were used extensively to deliver the course. Supplementary materials designed by the course development team for exploring multi-bases and studying ideas related to fractions rounded out the course materials. (Once the Assessing Teacher Leaders Math Understanding new editions for these two texts are made available, we may no longer need to insert these supplementary materials.) Course grades were based on a precourse assignment (described below), two tests (the second test was the posttest assessment instrument), problem sets, and a final reflection paper.

The course instructors used the DMI materials to deliver instruction. These materials include cases written by practicing teachers about their actual classroom experiences as their students' explored important mathematical concepts that underlie the elementary school curriculum. One of the key features in the cases is the focus on students' strategies as they build mathematical knowledge and develop computational and problem-solving skills.

Class meetings followed a seminar-like structure. Participants read assigned cases before coming to class. During class time, in small groups, they engaged in discussion as they addressed a series of questions (i.e., Focus Questions) about the mathematical ideas reflected in the cases. After meeting in small groups, they shared their ideas during follow-up whole class discussions in which the course instructors and participants further explored key ideas related to the cases including underlying mathematical ideas related to students' solution methods. During these discussions, participants unpacked a wide range of mathematical concepts covered in the K-5 mathematics curriculum as well as explored related pedagogical practices.

In addition to engaging in case discussions, participants completed mathematical activities that accompanied each set of cases. As they did so, they had opportunities to revisit some of underlying mathematical ideas related to the types of solution strategies that students used in the cases. They also had opportunities to develop their own mathematical understandings as they solved similar or related problems. As a consequence of participating in these types of activities, participants seemed to make connections between their personal experiences as mathematical learners and their instructional practices.

We suspect that one of the reasons that participants made shifts in their beliefs about mathematics teaching and learning was related to the kinds of experiences they had as they participated in class activities. They had opportunities to be learners of mathematics in a classroom setting in which they, along with the course instructors, co-created a learning environment that mirrored many of the reform-based scenarios highlighted in the cases. As a consequence, they experience first-hand, as learners, how the instructors used various instructional strategies to facilitate rich classroom discussions. So the very concepts and pedagogical skills that the participants uncovered through analyzing cases and engaging in mathematics explorations aligned with how they perceived themselves as learners in the course.

By aligning the course goals and instructional practices with the DMI developers' goals, the course instructors provide participants with opportunities to reflect on themselves and their students as learners as they used the course activities to explore these ideas.

To assess the participants' progress throughout the course, instructors used several different types of assessments: three problem sets, two tests and a final course reflection. The participants' performance on these assignments comprised their course grade. The instructors used traditional grading procedures to evaluate these assessments. In addition, the participants received full credit for completing the pretest assessment instrument. This score was also part of the raw score that was calculated to determine the participants' final course grades. Content included on the first test was taken exclusively from *Building Systems of Ten*. The second test, by design, was cumulative so that it could serve as the posttest (and pretest) assessment instrument; it covered materials from both DMI texts. The second test served as a cumulative assessment; instructors evaluated participants' responses using regular assessment procedures.

The second test also served as the posttest assessment; it was rated using the accompanying rubric to determine if participants made significant gains as a consequence of completing the course.

The problem sets were paper and pencil assessments that were used to evaluate the participants' understanding about specific big ideas addressed in the cases. Participants worked individually or in pairs to complete the problem sets. Each problem set typically consisted of 10 problems that required participants to practice or further explore ideas that were addressed (1) in the cases, (2) in the mathematics activities, or (3) that were highlighted during our class discussions. Some items addressed mathematical concepts; some addressed pedagogical content ideas.
Figure 1 contains three examples of the problems that are taken from different problem sets.

Insert Fig. 1 Sample Problems from Problem Sets
Problem 1 is an example of the types of explorations participants engaged in as they considered ideas related to our base ten system. For this problem, for instance, participants needed to explain how zero is not only a quantity, but also it's placement in a numeral signifies a position for some power of ten. These ideas were also addressed in one of the cases that the participants read in *Building Systems of Ten*. Problem 2 developed to highlight one of the ideas that surfaced in one of the cases in *Making Meaning for Operations*. As participants worked on this second problem, they might have considered ideas related to division, specifically how invented strategies might entail using the distributive property. For 216 ÷ 9, participants might provide as one example, the following string of equivalent expressions (210 + 6) ÷ 9 = [(210 + 6) ÷ 3] ÷ 3 = (70 + 2) ÷ 3 = 72 ÷ 3 = 24. Note that to divide 210 and 6 by 3, one is essentially employing the right distributive property for whole numbers. This task, unlike Problem 1, was an extension problem related to concepts students explored in some of the cases as they employed different invented strategies to solve division tasks. By exploring different ways to solve these types of division tasks, participants could examine the important role that the distributive property for division of whole numbers might play. They also had the opportunity to compare and contrast aspects of two related operations, multiplication and division.

Problem 3 (see Figure 1) provided participants' opportunities to explore some of the underlying concepts needed to understand the procedure for dividing by a fraction 'invert and multiply.' More importantly, participants could examine how the meaning of division needs to be redefined when working with fractions (cf., Ma, 1999). As in this example, for instance, they might consider how partitive division is extended when working with fractional divisors (less than 1). This concept is one of the core ideas addressed in the DMI cases and the mathematics activities that addressed the four operations. Through these types of problems (Problems 1, 2 and 3) and others, participants had opportunities to develop conceptual understandings about fundamental ideas that underpin mathematics in the elementary curriculum. In Problem 3, for instance, they might explore ways to construct a diagram that illustrated a partitive interpretation and to explore mathematical logic behind the traditional algorithm for division. More generally, by engaging in these types of activities, participants could explore important mathematical ideas as well as consider how they might use these ideas in their daily work with students. Many of the participants had never had these types of opportunities before enrolling in this course. We suspect that the opportunities that they had contributed in part to the progress that they made.

In this section, we have addressed some of the course activities to highlight the types of experiences participants had as they used the course materials and completed assignments. Our purpose in providing this background is to build a rationale for why we chose particular items for the assessment instrument. It was important to include items that were isomorphic per se. As part of our goal for developing the assessment instrument, we hoped that items would be representative of the important ideas that were addressed in the course. By doing so, we might align our assessment practices with our instructional goals without compromising one or the other."

Excerpted from Empson, S. B. (1999). ***Considerations of systemic change and teachers' knowledge of students' novel strategies for whole-number operations.*** **Paper presented at the annual meeting of the American Education Research Association, Montreal, Canada.**

"While curriculum materials cannot alone provide teachers the knowledge needed to teach in new ways, they can provide leverage for changes in practice and hence learning. A particular question is what kinds of learning and new knowledge about content and children's thinking can an innovative curriculum program motivate? In particular, what basis for making new knowledge claims about mathematics and about students might the teaching of such a curriculum provide teachers?...

Ball and Cohen (1996) have proposed that properly designed textbooks have the potential to help scale c change. Research on children's thinking about multidigit operations provides a backdrop against which to pose questions about the nature and extent of what teachers might learn as they teach using such materials. To consider this question, examples have been culled from a set of interviews with teachers about what they learned implementing Investigations in Number, Data, and Space (1995-1998, TERC). Participants consisted of all the teachers in grades three, four and five at a single elementary school, located in a district that has adopted Investigations as its elementary mathematics program."

Excerpted from Featherstone, H., Smith, S. P., Beasley, K., Corbin, D., & Shank, C. (1995). ***Expanding the equation: Learning mathematics through teaching in new ways.*** **East Lansing, MI: National Center for Research on Teacher Learning.**

"The Summer Math Program at Mt. Holyoke College has created the opportunity for in-service teachers to experience learning in the kind of classroom situation that the NCTM Standards asks them to provide for their students. The program gathers groups of practicing teachers who meet for two weeks in the summer to explore mathematics from elementary and secondary curricula. The teachers also interview children to probe their mathematical thinking, design a lesson based on that knowledge, and teach it. They explore the mathematics in groups of three or four. The teachers that Shifter and Fosnot (1993) describe experienced reduced levels of math anxiety in the supportive atmosphere of the groups. This relaxed atmosphere provided them an opportunity to think about mathematics without fear of evaluation. The members of the groups had opportunities to experience success in thinking about and solving mathematical problems…

The teachers are members of Investigating Mathematics Teaching (IMT), a project of the National Center for Research on Teacher Learning (NCRTL). This group of seven teachers and three researchers started meeting in the fall of 1991 to explore a multi-media collection of materials documenting teaching and learning of mathematics in two elementary math classes, one of which was taught by Deborah Ball.[1] During that fall, Helen Featherstone, Lauren Pfeiffer, and Stephen Smith structured activities around watching videotapes of Deborah Ball's third grade mathematics class and looking at Ball's journal and those of her students. They also visited the seven participating teachers' classrooms and interviewed each teacher on a regular basis. In January of 1992, the focus of discussions in the meetings began to move toward conversations around individual teachers' practices. The group has continued to meet on a bi-weekly basis during the school year. Helen, Lauren, and Steve continue to visit classrooms and interview teachers…

Although the other four teachers in the IMT group hail from four different districts, Debi, Kathy, and Carole all teach at the same urban elementary school. Carole and Kathy have been colleagues there since 1983; Debi is a relative newcomer, having been assigned to student teach in Kathy's second grade classroom in 1990 and having then stayed on in the school as a 'co-teacher' assigned, as part of Averill's Professional Development School effort,[2] to provide restructured time to a team of four primary grade teachers—a team that includes Kathy and Carole…

For Carole, writing her own number sentences, examining the number sentences that Ball's students had written in their math notebooks, listening to their discussion of these sentences, bringing the task to her own third graders, and observing what they did and said became both a pivotal event and a metaphor for changes in her own view of mathematics that grew out of observing and listening in a new way to her students as they grappled with math…

A few days later she gave her own third graders the same task; like Ball's students, they ranged adventurously across the numeric territory they knew…"

Excerpted from Franke, M. L., Carpenter, T., Fennema, E., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining, generative change in the context of professional development. *Teaching and Teacher Education, 14* (1), 67–80.

"*Workshop sessions.* The workshops during Year 1 and Year 2 focused teachers' attention on the research-based knowledge related to the development of children's mathematical thinking in addition/subtraction, multiplication/division, and place value, fractions and geometry (See Carpenter et al., 1996 for description of the workshop content). In working with the teachers, we attempted to build on their existing knowledge about children's thinking…

Therefore, we showed the teachers videos of children solving problems. We asked them to discuss what they had seen. Teachers shared their own students' solutions to an assigned problem like the ones seen in the video. Then teachers worked in small groups to sort the student work in terms of the development of the children's mathematical understanding. We consistently engaged the teachers in tasks that (a) helped them think about how their students would solve particular problems, (b) pushed them to discuss their thoughts so we could understand their thinking, and (c) helped them organize their own thinking about the development of children's thinking. (See Fennema et al., 1996 for a more complete description of the workshops).

Community interactions. The workshops with the teachers provided one avenue of teacher learning. The teachers also learned through their interactions with the students and their colleagues. We wanted the teachers engaged in CGI to have an opportunity to work together, both within and outside of their school communities. Therefore, in the overall project we worked with teachers from five schools. Ninety percent of the kindergarten through third-grade teachers at each school had agreed to participate. All of the teachers from the five sites participated in the workshops together. This arrangement created opportunities for teachers to work with colleagues within and across their grade level, both within and across schools. The decision to work with substantial numbers of teachers within each school also created pressure for some teachers to participate who may not have originally planned on participating…

Beyond working in workshops across schools we wanted to allow for ongoing support in teachers' classrooms and in discussions that occurred outside the classroom in their schools. Within each school, an experienced CGI teacher became the mentor teacher. The mentor teachers had a half-day release each week. The specific type of support varied depending on the mentor and the teacher, but included observing in the teacher's classroom and discussing the children's thinking, planning lessons together, and assessing children together. Yet, the mentor teacher was not the only source of support within the school; the teachers provided this support for each other as well. The teachers operated as sounding boards for each other as they thought about how to use knowledge of children's thinking in classroom practice.

Research staff. The final source of support for the teachers came from our more individual interactions with the teachers. A graduate student researcher interacted with each teacher; in most cases the same graduate student served all of the teachers in a given school. The project staff visited each teacher’s classroom informally on a regular basis. In Years 1 and 2 informal visits occurred on average once every two weeks. In Year 3, the visits occurred less frequently, on average once a month. During these visits the staff would take field notes and interact with the teacher regarding the mathematical thinking of the children in her classroom.”

Excerpted from Goldsmith, L. T., & Seago, N. (2007). *Tracking teachers' learning in professional development centered on classroom artifacts.* Paper presented at the Conference of the International Group for the Psychology of Mathematics Education, Seoul, Korea.

THE STUDY CONTEXT

"Data have been gathered in four research sites in the U.S.: two groups of teachers on the east coast who participated in Fostering Algebraic Thinking (AT) PD and two west coast districts participating in Video Cases for Mathematics Professional Development (VCM) PD. All of the groups were facilitated by the lead authors of the respective programs, (Driscoll for the AT groups and Seago for the VCM groups), ensuring high fidelity of implementation. Both seminars involved 12, three-hour sessions. Both PD programs involved 36 hours of PD (12, three-hour sessions). The VCM groups completed all 12 sessions in a single academic year (2003-2004) and the AT groups completed the PD over the course of three semesters (October 2003- January 2004). In all, 49 middle and high school teachers participated in the groups, 20 in the AT groups and 33 in the VCM groups. Sixteen teachers (four from each site) are being followed more closely to create case studies. Seminar participants included both veteran and early career teachers. Slightly more than a quarter of the teachers participating in the PD had been in the classroom for 5 years or fewer; the entire group of seminar participants averaged approximately 10 years of teaching. In addition to the 49 PD participants, 25 teachers served as a comparison group for our pre/post written measures. These comparison teachers came from the same districts as the PD participants."

Excerpted from Miller, L. D. (1991). Constructing pedagogical content knowledge from students' writing in secondary school. *Mathematics Education Research Journal, 3*(1), 30–44.

"Three secondary mathematics teachers from a large metropolitan high school in the United States agreed to participate in a study to examine the potential teacher benefits of using impromptu writing prompts in mathematics classes. This paper will focus on two of these teachers. One teacher was relatively inexperienced as a second year teacher; the other had more than fifteen years experience teaching secondary mathematics. They both taught first year algebra. Successful completion of first year algebra is a graduation requirement of the school district participating in this study. Generally, year 9 students (fifteen years old) enroll in first year algebra. One class of students (25–30 students per class) for each teacher participated in the study.

Students were generally asked to write at the beginning of a class unless the prompt asked about something that occurred in class that day. They were given five minutes to read the prompt, formulate their response and write it on paper provided by the teacher. Initially students were asked to write during three out of every four instructional periods. As the study progressed, the number of writing tasks were reduced to about three per week. Students were not rewarded for writing nor penalized for not writing. Thus, every student did not write every day.

Approximately fifty-five prompts were used in the study. Twenty were directed toward the affective domain; for example, attitudes toward and anxieties about mathematics, and feelings about class and topics being studied. Ten queried students' attitudes toward their responsibility in learning; that is, what determined whether or not they did their homework and how they prepared for an assessment. Twenty asked students to explain a mathematical concept, skill or generalization. Occasionally students were asked to write on a 'fun topic' or to write anything they wanted to write on a topic of their choice. One prompt in this category asked students to identify their favorite single digit number and to explain why they liked it. Another prompt said 'Write about any topic you want to today. Whatever you want to say, write it.'

Teachers generally read each set of writings the day they were collected. Once per week the teachers were asked to reflect upon what was being learned from reading the students' writings and to write about their predominant impressions. As the study progressed, the teachers' reflections became more evident in oral discussions than in their writing.

The students' and teachers' writings were collected fortnightly for one semester along with notes taken during the collaborative team meetings and during discussions with individual teachers. Since the focus of this discussion is on how writing prompts can expedite the development of teachers' pedagogical content knowledge, the remainder of this paper will concentrate on selected prompts which asked students to write about their understanding of mathematical content.

An interpretive research methodology (Erickson, 1986) was implemented in this study. Documentary materials consisted of students' responses to timed, in-class impromptu writing prompts, teachers' writings about what they were learning from the students' writings, and field notes derived from discussions with individual teachers and during meetings of the research team. As suggested by Glaser and Strauss (1967), a thorough analysis of the students' writings was completed. These writings were searched for recurring patterns that contributed to the identification and explanation of ways in which impromptu writing prompts enhanced the teachers' ability to assess how well students were comprehending the mathematics they were studying. In many respects, this was an exploratory study since very little research has been undertaken to examine benefits for teachers in using writing in mathematics classes. Thus, it was deemed inappropriate to search for conclusive empirical evidence which would allow the researchers to predict teacher benefits from using impromptu writing prompts in a mathematics class. More relevant to the research team was an investigation of what teachers can learn from reading their students responses to impromptu writing prompts."

Excerpted from Sowder, J. T., Phillip, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). ***Middle-grade teachers' mathematical knowledge and its relationship to instruction*****. Albany, NY: State University of New York Press.**

"A brief summary of the seminars over the 2-year period is provided so that the readers will have some understanding of the mathematical content to which the teachers were exposed and how they responded to the content.

An introductory meeting in the spring of 1992 was intended to acquaint the teachers with one another and to give them an indication of what to expect from the seminars, and to give us an indication of what to expect from the teachers. During the session, the teachers were given an excerpt from a lesson on fractions (Borko et al., 1992). In the ensuing discussion, one teacher said:

> Maybe I'm way off the wall, but I don't teach kids to flip numbers upside down.... So we review multiplying fractions….Then I put up a problem with division. (The teacher wrote a division problem on the board and drew two large Xs through the fractions while reciting the following.) I say, 'Follow these lines and multiply, and you got your answer. Just go 'I hate math; I hate math. Boy! Do I really hate math!' (See chapter 8 for more detail.)

The other teachers responded positively to this method. They felt that teaching fractions was extremely difficult; any 'gimmicks' would be useful. They indicated that they did not think it was possible to 'teach fractions with understanding,' and some also used Explorer calculators for multiplying and dividing fractions. They felt that students' demands for the answers prevented them from teaching conceptually.

Year 1. In the fall of 1992 we held 2 full-day seminars for the teachers. We began by discussing data on how children compare decimal numbers (from Resnick et al., 1989), then worked on place value with decimal numbers via the Blocks Microworld (P. Thompson, 1992). In the afternoon of the first day the teachers completed the Content Knowledge Assessment instrument. We made copies of the completed tests and discussed them among ourselves in terms of the areas on which we should focus our efforts. The tests were returned to the teachers unmarked, and the second day was devoted to discussing the items on that test. The teachers became very involved in considering their own responses and those of the others, then thinking about how their students would react to some of the items. Some of the items had been used with students (Armstrong & Larson, 1995), and the ways in which students thought about those items and solved them were discussed with the teachers. (When relevant, each teacher's work on this assessment is discussed in the individual case studies.)

For the remainder of the year, approximately half the seminars were presentation-focused—that is, a researcher prepared a presentation based on research with children. The presentations were informal, and there were questions and discussions throughout the presentations. When the presentations were made by visiting researchers from outside the university, a few additional teachers were invited, so that the audience was approximately a dozen. (The presentation-based seminars were substantially the same as the written versions of the presentations appearing in *Providing a Foundation for Teaching Mathematics in the Middle Grades* [J. Sowder & Schappelle, 1995] as chapters by Armstrong & Bezuk, Harel, Kieren, Lamon, Mack, J. Sowder, L. Sowder, and P. Thompson.) The remaining seminars focused on follow-up discussions of these presentations, on discussions on topics selected by the investigators on the bases of their knowledge of the teachers' content understanding, of results of tests and interviews of the students of the teachers, and of questions raised by the teachers. (A more detailed presentation of the teacher interactions and struggles to understand the content of these seminars is presented in chapter 4 and also in J. Sowder & Philipp, 1995.)

The first two seminars were intended to lead the teachers to see the value of sense-making as part of the enterprise of teaching. A presentation on rational number sense led to practice with mental computation and estimation and to examination of sense-making with operations and algorithms.

The next four seminars focused on developing the teacher's understanding of fractions and fraction operations. Presentations by Mack, Armstrong, Bezuk, and Kieren provided the teachers with research-based ways of presenting critical ideas about fractions and fraction operations. Examples of students' thinking and working with fractions challenged the teachers to think about their role in teaching fractions in meaningful ways. The fourth seminar was devoted to discussion of the results of the teachers' students' work on the Fraction Understanding Test (provided in Appendix F). The items tested for conceptual understanding rather than algorithmic skill. The teachers were surprised and distressed with the results. Although they recognized that they were not responsible for the poor performance (the tests were administered after students had been in their classes for less than 2 months), they also realized that until this seminar they had little comprehension of what their students knew and did not know, thus making it difficult for them to base instruction on students' knowledge. (This seminar was summarized and analyzed in Armstrong, Philipp, & J. Sowder, 1993.)

A more holistic look at both whole number and rational number operations was the subject of the presentation 'Addressing the Story-Problem Problem' by L. Sowder. He discussed the connections between the operations and the real-world applications, focusing on what elements in a situation lead to choosing the correct operation.

The next three seminars were informal; they focused on critical incidents in the teachers' own classrooms and on discussion of the previous presentations. The teachers compared ways that their own planning for instruction on fractions was changing.

In the two following seminars we turned to the topic of proportional reasoning; the discussion was based on a presentation by Lamon. Proportional reasoning as multiplicative reasoning was discussed in some detail. These seminars led into Harel's presentation in which he outlined students' progress through additive reasoning into multiplicative reasoning. The final presentation of the year, by P. Thompson, focused on quantitative reasoning in both simple and complex situations.

For the closing seminar of the first year we chose several transcript excerpts from the seminars during which teachers had struggled with mathematical concepts and had finally came to a deep understanding of them. The teachers were given the assignment of reading excerpts and providing written reactions to them at a later date. To set the stage for this assignment, the investigators each earlier wrote reflections on the year's work and shared them with the teachers at this seminar. For the remainder of the seminar, the teachers talked informally about what they had learned and how they had changed over the course of the year. The conversations focused on the seminars, our classroom observations, their own planning and insights, and their classroom interactions with students.

Year 2. During Year 2, several of the topics introduced during the Year-1 seminars were revisited, sometimes through discussions of (sometimes videotaped) segments of the participating teachers' classroom rational number lessons that had been observed by the researchers and sometimes through revising the papers written by the presenters of Year 1. The first seminar of the year was devoted to eliciting individual teacher reflections, partly to determine ways to provide seminars of most benefit to the teachers at this stage in their participation in the project. Teachers spoke about their mathematica1 goals for the year, their mathematical expectations for their students for the year, their roles as teachers, perceived obstacles in teaching mathematics, the growth of students from additive to multiplicative reasoning, changes they were making or would like to make in their mathematics teaching, and what each hoped to gain from the project during the coming year.

In seminar discussions of observed teachers' classroom lessons (sometimes with videotaped segments presented), the importance of consistently relating the part to the unit was an issue in both the fractions and decimal lessons being discussed; all of the teachers recognized this as an issue in their own classrooms. Issues related to the use of models for rational numbers also arose in these seminars. Before one seminar, two researchers had visited the same teacher a few days apart; the second had the opportunity to see implemented the first's suggestion to incorporate proportional reasoning into a lesson. Describing this lesson sparked a discussion of teachable moments—awareness of situations in which opportunities to develop important ideas, in this case proportional reasoning, arise.

The one topic tested on the initial Content Knowledge Test but not addressed during Year 1 was that of weighted average in rate problems. This difficult topic was approached in Year 2 through the use of P. Thompson's *Over and Back* (1994) microworld.

Just as in Year 1, most of one seminar was devoted to discussion of students' fraction-understanding-test and interview results. The teachers appreciated the limitations of the pencil-and-paper instrument, even though it focused on conceptual learning and the greater richness of the responses in interviews in which answers could be probed for reasoning and in which misinterpretations of the problems were evident.

During these Year-2 seminars, even more than in the Year-l meetings, the teachers often raised questions or shared classroom experiences that led to extended discussions (e.g., Darota gave students a problem to do individually so that she could work on report cards, but the problem instead turned into an extended lesson on ratio). Issues about standardized tests and textbooks were raised repeatedly. The importance of the teachers' having deep understanding of the content, the big ideas within a topic, the connections among topics—instead of merely presenting interesting problems that are not necessarily part of a bigger, overall picture of rational numbers—was recognized by the teachers and was raised by them more than once during the Year-2 seminars.

Year 2 concluded with a seminar in which we, the researchers, explained that we would now be trying to tell what had been learned from this project, and the teachers were asked to reflect on their participation and to tell what had been learned from their points of view. The teachers spoke quite passionately about how much they had learned and about the need for all teachers to have more opportunities to focus on mathematics during professional development."

Excerpted from Stecher, B. M. & Mitchell, K. J. (1995). ***Vermont teachers' understanding of mathematical problem solving and "good" math problems.*** **Paper presented at the annual meetings of the American Educational Research Association, San Francisco, CA.**

"A two-stage process was used to select a representative sample of 20 fourth grade teachers who were using math portfolios during the 1993-94 school year. In the first stage, the population of Vermont schools was divided into four groups based on mean 1992-93 uniform test scores; a stratified random sample of 20 elementary schools was then drawn.[1] Two schools with fewer than ten fourth grade students were replaced by others drawn at random from the same strata.

In the second stage, one fourth grade teacher was drawn at random from each school and invited by letter to participate in the study. Neither district administrators nor school principals were notified about the study or of the teachers who were participating. During initial conversations, two of the 20 teachers were removed from the sample because they had not been teaching for the full school year or were not using portfolios in mathematics. Three other teachers declined to participate. Each of the five was replaced at random by another fourth grade teacher from the same school (if there was one) or by sampling another school from the same stratum (if there was no other fourth grade teacher in the school).[2]

Participating teachers had between one and four years of experience with portfolios (four years was possible if a teacher participated in the portfolio pilot in 1990-91) and between one and 40 years of teaching experience. On average teachers in the study had ten years of teaching experience, spending about one-half of this time at the fourth grade level. Figure 2.1 shows the distribution of teaching experience in the sample, and Figure 2.2 shows the distribution of portfolio experience….

The Vermont Department of Education offered a summer training institute and four subsequent training workshops during the school year. All of the teachers in the sample attended at least one training session during the 1993-94 school year. They also attended training sessions the previous year. Over the past two years, one-quarter of the teachers attended some of the training sessions offered, 40% attended most of the sessions, and 35% attended all training sessions….

These common tasks provided a basis for quantifying variations among teachers on a number of dimensions. For example, teachers were asked to evaluate and suggest improvements to two specific tasks taken from (or patterned after) those in Vermont portfolios.[4] Teachers also were shown two pairs of portfolio tasks addressing similar mathematical topics. They were asked to explain which member of each pair would be better as an instructional activity and which would be more likely to produce high-scoring best pieces.

The subsequent telephone interviews focused on teachers' understanding of problem solving, task selection, and portfolio-related instruction. Questions also were asked about the portfolio tasks that appeared in the written survey. These questions examined the problem-solving skills students would use in response to the tasks and teachers' judgments about the merits of the tasks."

Excerpt (a) from Swafford, J. O., Jones, G. A., Thornton, C. A., Stump, S. L., & Miller, D. R. (1999). The impact on instructional practice of a teacher change model. ***Journal of Research and Development in Education, 32*****(2), 69–82.**

Excerpt (b) from Swafford, J. O.; Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. ***Journal for Research in Mathematics Education, 28*****(4), 467-83.**

(a)

Participants in the Intervention

"The subjects were 48 middle-grades (4-8) teachers participating in Project LINCS. These participants, all volunteers, came from 30 schools and 12 school districts within a 50-mile radius of a Midwestern university. Pairs of participants from the same school were actively sought, but only 32 came from schools with 2 or more participants. Their mean number of years of teaching experience was 13.6….

Intervention Program

Project LINCS was a 3-year intervention program designed to enhance teachers' knowledge through annual 4-week summer content courses, accompanying 8-hour research seminars on student cognition, and 6 half-day seminars during the academic year focusing on pedagogical practice. The intervention also incorporated structured on-going teacher collaboration and reflection.

Content courses. The summer content courses addressed probability and statistics in year 1, geometry in year 2, and algebra in year 3. The probability and statistics course emphasized the exploration of data and the use of simulation to determine probabilities. Visual displays and descriptive statistics were used to examine characteristics and patterns in data; and theoretical probabilities, simulations, and data analysis were used to solve a wide variety of probability problems. The geometry course focused on the exploration of two- and three-dimensional shapes using the van Hiele (1959/1985) levels of recognition, analysis, and informal deduction as a basis for instruction. In particular, the course incorporated an investigation of polygons and their properties; tessellations; polyhedra and their properties; length, area, and volume measures; and motion geometry. The algebra course explored families of functions in problem contexts. This exploration used graphical, tabular, and symbolic representations to investigate linear, quadratic, and exponential functions. The instructional approach adopted in each course can be described as 'teaching via problem solving' (Schroeder & Lester, 1989) and modeled the pedagogy advocated in the half-day seminars. Computers and graphics calculators were used in all courses.

Research seminars on student cognition. The companion research seminar reviewed and discussed research findings on students' cognition in each of the three content areas and reflected on the implications of these for classroom instruction. The seminars examined the research on the development of probabilistic thinking (Shaughnessy, 1992); van Hiele levels (Fuys, Geddes. & Tischler, 1988); and students' understanding of variables and their uses (Kieran & Chalouth, 1993). Each year, participants also interviewed a student at their grade level in order to evaluate the student's thinking with respect to that summer's content topic.

Seminars on pedagogical practice. During each of the 3 academic years, participants attended 6 half-day seminars. The seminars analyzed practices advocated in the *Teaching Standards*

(NCTM, 1991). Topics included alternative assessment, cooperative groups, classroom discourse, worthwhile mathematical tasks, and writing in mathematics. These practices were discussed in the seminars, and suggestions for their implementation were presented.

Collaboration and reflection. Each seminar had a formal segment where participants shared ideas and successful practices as well as informal opportunities for sharing during breaks and activities. Also, as part of each district's contribution, participants were given a half-day per semester for collaboration within their building.

Each year participants videotaped and analyzed two classroom lessons. They also kept a reflective journal, and at the end of each year they provided a summary of their journals for project staff. This summary highlighted changes that had occurred in their teaching and identified their goals for the coming year. The final journal summary reflected on the entire 3 years and discussed how their instructional practice had been influenced by the project."

(b)

Subjects

"The subjects were 49 middle-grade (4-8) teachers participating in Project LINCS, a 3-year professional development project funded by the National Science Foundation. The participants, all volunteers, were drawn from a commuting distance of a midwestern university and received 5 semester hours of graduate credit and a summer stipend for each year of participation. They were divided into two sections for the summer courses. Group 1 comprised the Grades 4-5 teachers and Group 2 the Grades 6-8 teachers….

Intervention Program

Each year of Project LINCS consisted of a 4-week summer session and six half-day seminars during the academic year. Each summer the program emphasized a different subject-matter content, with geometry being the focus area during the second summer. The geometry program consisted of a mathematics content course on geometry, which met 3 hours a day for 4 days a week, and a research seminar, which met for 2 hours once each week. The two sections of the content course were taught by two mathematics education faculty, one of whom was the second author. The research seminar was conducted by the first author. The third author coordinated the academic-year seminars.

The geometry course focused on the exploration of two- and three-dimensional shapes through recognition, analysis, and informal deduction, with greater emphasis on analysis and informal deduction. Instructors adopted an instructional approach that has been described as 'teaching via problem solving' (Schroeder & Lester, 1989). Using this approach, each session commenced with the presentation of a problem that embodied key aspects of the topic. Participants worked on the problem in small groups and then shared their solutions in a class discussion. During the class discussion, solution strategies were refined, extension problems were formulated and solved, connections were identified, and discussions ensued on the van Hiele level of key tasks associated with the problem. The textbook for the course, *Geometry: An Investigative Approach* (O'Daffer & Clemens, 1992), was compatible with the 'teaching via problem solving' approach. As part of the course, the participants also developed an instructional unit and accompanying assessment plan for their respective grade levels.

The research seminar presented the van Hiele theory of cognitive development and instruction in geometry. This was followed by an examination of the research on the van Hiele levels of students (Burger & Shaughnessy, 1986; Fuys et al., 1988; Mayberry, 1983; Senk, 1989). Research on geometry text materials (Fuys et al., 1988; Whitman & Komenaka, 1990) and the results of national and state assessments in geometry were also reviewed. In addition to the research readings, teachers had the option of either interviewing a student at the grade level they teach or analyzing instructional activities in their textbooks by van Hiele levels. For the student interviews, 36 items developed by Mayberry (1981) for assessing the first three van Hiele levels were used. The textbook analysis used the methodology described in Fuys et al. (1988). Both activities were designed to help make the research real to the participants by linking the results and methodology either to one of their own students or to their own textbooks. The interviews also served to give the teachers greater insight into their students' thinking about geometry."