Why Teachers' Mathematics Content Knowledge Matters: A Summary of Studies

Professional learning opportunities for teachers of mathematics have increasingly focused on deepening teachers' content knowledge. Based on a number of research studies identified in a large-scale literature review, teachers' mathematics content knowledge makes a difference in their professional practice and their students' achievement. Additional information on how these studies were identified and reviewed, and a summary of the methodology, can be found at:

http://www.mspkmd.net/index.php?page=01_4a-3b2

Teachers' Mathematics Content Knowledge Influences Their Professional Practice

A systematic search of education research databases surfaced a total of 35 studies examining the link between teachers' mathematics content knowledge and their professional practice. Across this set of studies, several different aspects of professional practice were investigated. The largest set of studies examined the relationship between either teachers' disciplinary mathematics content knowledge or their knowledge of student thinking about mathematics and their classroom instructional practice. A smaller set of studies investigated the relationship between teachers' disciplinary mathematics content knowledge and their implementation of mathematics curriculum materials. A few studies examined the relationship of teachers' disciplinary content knowledge and their professional community building within their schools. The following sections, organized by the aspect of practice each investigated, describe the studies and their findings.

Teachers' Disciplinary Mathematics Content Knowledge is Related to Their Classroom Instructional Practice

Twenty-two studies were identified that examined the influence of teachers' disciplinary mathematics content knowledge on their instructional planning and classroom practice. Several of these studies included multiple sub-studies. Information about the research studies is displayed in Table 1.

Nine of the studies focused on elementary school teachers,[1] one on both elementary and middle grades teachers,[2] five on middle grades teachers,[3] and six on high school teachers.[4] One study[5]

---

[1] Chi-chung, Yun-peng, & Ngai-ying, 1999; *Heaton, 1992*; Lehrer & Franke, 1992; Leung & Park, 2002; Ma, 1999; *Prawat, 1992; Prawat, Remillard, Putnam, & Heaton, 1992; Putnam, 1992; Putnam, Heaton, Prawat, & Remillard 1992; Remillard, 1992*; Schwartz & Reidesel, 1994; Spillane, 2000; Stein, Baxter, & Leinhardt, 1990; Wilkins, 2002. (References in italics report on components of a single study.)

[2] Hill et al., 2008.

[3] Cai, 2005; Iszak, 2008; Sowder, Phillip, Armstrong, & Schappelle, 1998; Thompson & Thompson, 1996; Thompson & Thompson, 1994; Wilson, 1994.

[4] Chazan, Yerushalmy, & Leikin, 2008; Chinnapan & Thomas, 1999; Heid, Blume, Zbiek, & Edwards, 1999; Llinares, 2000; Vale & McAndrew, 2008; Zbiek, 1995.

involved both middle grades and high school teachers. Because more studies were conducted in the elementary grades, the findings are more strongly supported at this level, but a growing number of studies in recent years have examined middle and secondary grades teachers. Many of the studies included observations, audio taping, and videotaping, often of multiple lessons. Interviews of the teachers were also used in a number of the studies to provide additional data and to substantiate the relationships between knowledge and practice the researchers discerned.

All but two studies reported a direct link between teacher content knowledge and teaching practice. For example, in a comparison of two mathematics teachers, one study[6] found that the more knowledgeable teacher presented problems in contexts that were familiar to the children in the classroom and linked them to activities they had previously completed. Another study related measures of ten elementary and middle grades teachers' mathematics knowledge for teaching to several aspects of their practice. The researchers reported that teachers with stronger content knowledge were more likely to respond to students' mathematical ideas appropriately, and to make fewer mathematical or language errors during instruction. The majority of studies found that the converse of this relationship also appears to be true; that is, a *lack* of content knowledge seems to limit a teacher's instruction.[7] For example, one study found that when teachers with weak content knowledge departed from their instructional materials, they tended to obscure or distort the mathematics concepts the students were expected to learn because they chose to augment instruction with inappropriate mathematical representations.[8] Of the two studies that did not find a direct link between teacher content knowledge and instructional practice, one[9] found that greater content knowledge strengthened the relationship between positive beliefs about standards-based teaching practices and reported use of these practices, and the other[10] found that teachers' conceptual understanding was not linked to conceptually-oriented teaching strategies.

Several limitations were identified in the methodologies of the studies. Although all of the studies had measures of teacher content knowledge, only four[11] included a quantitative measure that could be used readily in replications or extensions of the studies. In many cases, the measurement of teacher content knowledge was not fully described. Several studies used the same form of measurement to examine both teaching content knowledge and teaching practice, possibly leading to a confounding of independent and dependent variables.[12] In two of these

---

[5] Fernandez, 1997.

[6] Lehrer & Franke, 1992.

[7] Heaton, 1992; Heid, et al., 1999; Hill et al., 2008; Lehrer & Franke, 1992; Prawat, 1992; Prawat et al., 1992; Putnam, 1992; Putnam et al., 1992; Remillard, 1992; Spillane, 2000; Stein et al., 1990.

[8] Heaton, 1992.

[9] Schwartz & Riedesel, 1994.

[10] Leung & Park, 2002.

[11] Hill et al., 2008; Schwartz & Reidesel, 1994; Sowder et al., 1998; Wilkins, 2002.

[12] Chinnapan & Thomas, 1999; Fernandez, 1997; Iszak, 2008; Llinares, 2000; Spillane, 2000; Wilson, 1994; Zbiek, 1995.

studies,[13] each of which focused on only one teacher, the measure used to assess content knowledge might be idiosyncratic to the teacher involved in the study, making it difficult to consider generalizability of the results or replication of the study.

For two of the quantitative studies,[14] the measure of classroom practice was a self-report survey. This approach raises the concern that the measure may be capturing beliefs about teaching, rather than actual teaching practice. Also, the instruments used for measuring classroom practices in many of the studies either had low reliability or were not described in sufficient detail to communicate what was being measured.

There were also limitations to the conclusions of many of the studies. For example, most did not account for factors other than teacher content knowledge that may have influenced classroom instruction, although most of the studies recognized other possible influences. As noted above, one study[15] empirically demonstrated an interaction between content knowledge and beliefs about standards-based teaching practices as factors related to teachers' instruction.

Because a primary goal of most of these studies was intensive analysis to describe both the teachers' knowledge and practice in order to detail the connections between them, most of the studies of the relationship between teachers' disciplinary content knowledge and their teaching practice were conducted with fewer than five teachers.[16] Combined with the narrow contexts of the studies, these small sample sizes potentially weaken the generalizability of the findings. However, in several cases the extensive and deep data collection techniques suggest that the results may be broadly applicable. For example, a coordinated set of case studies of fifth grade teachers[17] sought subjects who represented a variety of contexts and found that in each case there was evidence that could illustrate the important relationship between teacher content knowledge and instruction. The argument for generalizability of this finding is that similar examples illustrating this relationship were found for each of the different teachers and contexts studied.

---

[13] Chinnapan & Thomas, 1999; Lehrer & Franke, 1992.

[14] Schwartz & Reidesel, 1994; Wilkins, 2002.

[15] Schwartz & Riedesel, 1994.

[16] Chazan et al., 2008; Chinnapan & Thomas, 1999; Heaton, 1992; Heid et al., 1999; Iszak, 2008; Lehrer & Franke, 1992; Llinares, 2000; Prawat, 1992; Prawat et al., 1992; Putnam, 1992; Putnam et al., 1992; Remillard, 1992; Spillane, 2000; Stein et al., 1990; Thompson & Thompson, 1996; Thompson & Thompson, 1994; Wilson, 1994; Zbiek, 1995.

[17] Heaton, 1992; Prawat, 1992; Prawat et al., 1992; Putnam, 1992; Putnam et al., 1992; Remillard, 1992.

Table 1
Studies Examining the Relationship of Teachers' Disciplinary Mathematics Content Knowledge and Their Classroom Instructional Practice

Name of Study	Grade Level	Number of Teachers	Content Strand(s)				Type(s) of Knowledge			Measure(s) of Teacher Content Knowledge			
			Number and operations	Disciplinary Content	Ways of Knowing	Pedagogical Content	Algebra	Measurement	Various	Assessments	Interviews	Observations	Other approach
U.S. and Chinese teachers' constructing, knowing, and evaluating representations to teach mathematics (Cai, 2005)	6–8	20	•	•		•					•		•
An analytic conception of equation and teachers' views of school algebra (Chazan et al., 2008)	6–12	3											
Teacher development, not accountability control, is the key to successful curriculum implementation: A case study of two primary schools in northeast China (Chi-chung et al., 1999)	K–5	2 schools		•		•			•		•		
Conceptual modeling of functions by an experienced teacher (Chinnapan & Thomas, 1999)	9–12	1		•			•				•		
The "*Standards*-like" role of teachers' mathematical knowledge in responding to unanticipated student observations (Fernandez, 1997)	6–12	9		•		•			•				•
Case studies of four fifth grade teachers (Heaton, 1992; Prawat, 1992; Prawat et al., 1992; Putnam, 1992; Putnam et al., 1992; Remillard, 1992)	2, 5	4	•	•	•	•	•		•		•		
Factors that influence teachers learning to do interviews to understand students' mathematical understandings (Heid et al., 1999)	9–12	3		•			•				•		

Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. (Hill et al., 2008)	2–6	10											
Mathematical knowledge for teaching fraction multiplication (Izsak, 2008)	6	2											
Applying personal construct psychology to the study of teachers' knowledge of fractions (Lehrer & Franke, 1992)	2, 5	2	•	•		•					•		
Competent students, competent teachers? (Leung & Park, 2002)	K–5	18	•	•		•		•			•		
Secondary school mathematics teachers' professional knowledge: a case from the teaching of the concept of function (Llinares, 2000)	9–12	1		•		•	•				•		
Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States (Ma, 1999)	K–5	95	•	•		•		•			•		
The relationship between teachers' knowledge and beliefs and the teaching of elementary mathematics (Schwartz & Reidesel, 1994)	K–5	140		•					•	•			
Middle-grade teachers' mathematical knowledge and its relationship to instruction (Sowder et al., 1998)	6–8	5	•	•		•				•	•		•
A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter (Spillane, 2000)	5	1		•		•			•		•	•	
Subject-matter knowledge and elementary instruction: A case form functions and graphing (Stein et al., 1990)	5	1		•		•	•				•	•	
Talking about rates conceptually, part I: A teacher's struggle (Thompson & Thompson, 1994) Talking about rates conceptually, part II: Mathematical knowledge for teaching (Thompson & Thompson, 1996)	6–8	1	•	•		•		•		•			
Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications (Vale & McAndrew, 2008)	7–11	3											
The impact of teachers' content knowledge and attitudes on instructional beliefs and practices (Wilkins, 2002)	K–5	407		•					•	•			
Implications for teaching of one middle school mathematics teacher's understanding of fractions (Wilson, 1994)	6–8	1	•	•	•	•					•		

Her math, their math: An in-service teacher's growing understanding of mathematics and technology and her secondary students' algebra experience (Zbiek, 1995)	9–12	1		•		•	•					•	•

Teachers' Knowledge of Student Thinking about Mathematics is Related to Their Classroom Instructional Practice

Nine studies focused on teachers' knowledge of student thinking about mathematics content in relation to their instructional planning and classroom practice.[18] The empirical knowledge base on this relationship is somewhat limited because seven[19] of these nine studies were conducted with elementary grades teachers, one of which also included middle grades teachers.[20] All but two of these[21] seven studies investigated teachers who had been involved in professional development opportunities based on *Cognitively Guided Instruction*. One study investigated this relationship with middle grades teachers,[22] and one with high school teachers.[23] Information about the research studies is displayed in Table 2.

Combining the studies by Fennema and colleagues[24] and those conducted by Bright and colleagues, Lubinski, and Warfield[25] provides a set of investigations focused on *Cognitively Guided Instruction* that includes both individual case studies and somewhat larger scale studies. The findings in these studies fairly consistently support the conclusion that teachers' knowledge of student thinking about mathematics has important influences on their practice. These findings, however, are limited in important ways due to the nature of the studies.

The findings of the larger scale studies were consistent with the theory on which the *Cognitively Guided Instruction* program was based, namely that teachers who gain research-based knowledge about student thinking in mathematics will adopt more cognitively-guided approaches in their practice. There are, however, limitations in the instrumentation and analyses that should be understood. Several of these instruments lacked variability in teachers' scores as most of the teachers scored quite high on them, so they could not be used effectively in analysis of the relationship of teachers' content knowledge and their practice. The one instrument that was used to support analyses of this relationship was used without correction for chance, despite the fact that responses were dichotomous. Also, multiple univariate comparisons were made without adjustment of the error rate for the number of comparisons. For these reasons, the findings of these studies should be treated as tentative, although consistency of the findings across studies strengthens confidence in them

---

[18] Anders, 1995; Bright, Bowman, & Vacc, 1998; Fennema et al., 1996; Fennema et al., 1993; Hill et al., 2008; Iszak, 2008; Lubinski, 1993; Vale & McAndrew, 2008; Warfield, 2001.

[19] Anders, 1995; Bright et al., 1998; Fennema et al., 1996; Fennema et al., 1993; Lubinski, 1993; Warfield, 2001.

[20] Hill et al., 2008.

[21] Anders, 1995; Hill et al., 2008.

[22] Iszak, 2008.

[23] Vale & McAndrew, 2008.

[24] Fennema et al., 1996; Fennema et al., 1993.

[25] Bright et al., 1998; Lubinski ,1993; Warfield 2001.

In the case studies, several illustrations of teaching practice of the studied teachers are linked to their knowledge of student thinking about content, both generally and in relation to specific students. In both cases, the research was conducted after the teachers had participated in *Cognitively Guided Instruction* professional development workshops. Contrasting examples of practice from these teachers prior to their professional development experiences, from other teachers, or from content areas not addressed in the professional development were not presented. It is consequently somewhat difficult to judge the nature and strength of the relationship of the teachers' knowledge of student thinking and their practice.

An important finding in this set of studies is that teachers who have experienced *Cognitively Guided Instruction* continued to use multiple frameworks to interpret teaching practice, and the frameworks they used shifted over time but not uniformly toward any particular framework.[26] Although the researchers did not seek an explanatory mechanism, and despite some unknown characteristics of the sample, this finding indicates that the influence of teachers' knowledge of student cognition in mathematics does not necessarily translate in predictable ways to practice.

One study in this group that did not involve *Cognitively Guided Instruction,* conducted by Anders,[27] presents a contrast in mathematics teaching practice for the same teacher with two different groups of students. The study introduces an approach for examining the relationship of a teacher's knowledge and practice through the concept of a "script." The use of the framework connects the teacher's knowledge of student thinking to the instructional practice in the two classes. However, it also reveals that the teachers' practice in the two classes was quite different in many respects, but did not identify whether these differences in instruction were related to any differences in the teacher's knowledge of the students' thinking in these two classes.

[26] Bright et al., 1998.

[27] Anders, 1995.

Table 2
Studies Examining the Relationship of Teachers' Knowledge of Student Thinking about Mathematics and Their Classroom Instructional Practice

			Content Strand(s)				Type(s) of Knowledge			Measure(s) of Teacher Content Knowledge			
Name of Study	Grade Level	Number of Teachers	Number and operations	Disciplinary Content	Ways of Knowing	Pedagogical Content	Algebra	Measurement	Various	Assessments	Interviews	Observations	Other approach
A teacher's knowledge as classroom script (Anders, 1995)	2–3	1				•			•		•	•	•
Teachers' frameworks for understanding children's mathematical thinking (Bright et al., 1998)	K–4	20	•			•						•	
A longitudinal study of learning to use children's thinking in mathematics instruction (Fennema et al., 1996)	1–3	21	•	•		•					•		
Using children's mathematical knowledge in instruction (Fennema et al., 1993)	1	1	•	•		•					•		
Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. (Hill et al., 2008)	2-6	10											
Mathematical knowledge for teaching fraction multiplication (Izsak, 2008)	6	2											
More effective teaching in mathematics (Lubinski, 1993)	1	1	•			•						•	•
Teaching kindergarten children to solve word problems (Warfield, 2001)	K	1	•			•					•	•	
Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications (Vale & McAndrew, 2008)	7-11	3											

Teachers' Disciplinary Mathematics Content Knowledge is Related to Their Implementation of Mathematics Curriculum Materials

The literature review identified three studies that focused on the effects of teachers' disciplinary mathematics content knowledge on the implementation of curriculum materials. One multi-part study was conducted at the middle grades level,[28] and two studies, one with multiple parts, were conducted at the high school level.[29] None of the studies focused at the elementary level, a clear gap in the empirical knowledge base. Information about these research studies is displayed in Table 3.

Manouchehri and Goodman's[30] investigations of curriculum implementation in the middle grades make important contributions because multiple curriculum materials were being implemented across the project being studied, and the two reports taken together offer both case studies and a larger-scale investigation of the influence of teachers' mathematics knowledge on their curriculum implementation. The case studies provide a contrast of a teacher whose stronger mathematics knowledge supports more complete implementation than that of a teacher with weaker mathematics knowledge. These teachers were matched on a number of important characteristics—grade level, experience, school, and initial enthusiasm for the new curriculum materials; one unexamined contextual condition was the characteristics of the students each teacher taught, which if they differed could have accounted for some of the differences in implementation. Overall, the contrasting cases provided appropriate evidence, carefully analyzed and from different sources, that the teachers' knowledge was a key factor in their differences in implementation. The larger study provides some support for the same findings, and the examples offered support those findings, but the representativeness of those examples compared to the entire dataset is not well-established. It is therefore not clear how widespread and consistent were the connections between teachers' content knowledge and particular patterns of curriculum implementation.

Lloyd and Wilson's[31] two-part study of one high school teacher's curriculum implementation was strong in that it followed the teacher over two years, documenting changes in practice that could be related to changes in the teacher's content knowledge over time. The study is somewhat unusual in that the teacher's content knowledge was documented to be fairly strong at the outset, although one slight, but important, weakness was established. The multiple examples provided in the first year study demonstrate how both the teacher's strong content knowledge, and the one documented limitation, played out in his implementation of the curriculum, although not in a way that appeared particularly detrimental to the lessons. In the second year study, changes in the teacher's practice are illustrated, and these changes mainly address the originally-identified limitation in the teacher's knowledge. The study did not measure the teacher's knowledge in the second year, so changes in his observed practice and how he described his instructional intentions and decisions were used as evidence of changes in his knowledge. The teacher was

[28] Manouchehri, 1998; Manouchehri & Goodman, 1998; Manouchehri & Goodman, 2000.

[29] Lloyd, 2002; Lloyd & Wilson, 1998; Sherin, 2002.

[30] Manouchehri, 1998; Manouchehri & Goodman, 1998; Manouchelri & Goodman, 2000.

[31] Lloyd, 2002; Lloyd & Wilson, 1998.

participating in a field test of the instructional materials, which included meetings with other teachers, so other factors may have accounted for some of the changes in his second-year instruction. Even without comparable measurement, a good case was made that more flexible knowledge of the content was involved in the second year implementation.

The other study of high school teachers[32] investigated how five teachers implemented new instructional materials. The study found that the teachers accessed and used their mathematics content knowledge in three different ways: (1) altering the mathematical aspects of the lessons to fit their own prior understandings of the mathematics content rather than following the intent of the curriculum developers; (2) teaching aspects of the lessons as designed in ways that differed from, and extended, their own prior mathematics content knowledge; and (3) using new content knowledge developed as a product of implementing the lessons to direct the course of the lesson in ways that differed from what was explicitly intended by the developers.

---

[32] Sherin, 2002.

Table 3
Studies Examining the Relationship of Teachers' Disciplinary Mathematics Content Knowledge and Their Implementation of Mathematics Curriculum Materials

Name of Study	Grade Level	Number of Teachers	Content Strand(s)				Type(s) of Knowledge			Measure(s) of Teacher Content Knowledge			
			Number and operations	Disciplinary Content	Ways of Knowing	Pedagogical Content	Algebra	Measurement	Various	Assessments	Interviews	Observations	Other approach
Reform-oriented curriculum implementation as a context for teacher development: An illustration from one mathematics teacher's experience (Lloyd, 2002) Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum (Lloyd & Wilson, 1998)	9–12	1		•			•				•		
Mathematics curriculum reform and teachers: What are the dilemmas? (Manouchehri, 1998) Mathematics curriculum reform and teachers: Understanding the connections (Manouchehri & Goodman, 1998)	6–7	66		•		•			•		•	•	
Implementing mathematics reform: The challenge within (Manouchehri & Goodman, 2000)	7	2		•		•			•		•	•	
When teaching becomes learning (Sherin, 2002)	9–12	5		•		•	•				•		

Teachers' Disciplinary Mathematics Content Knowledge is Related to Their Professional Community Building

Two studies examining teachers' disciplinary content knowledge in relation to their professional community building outside of the classroom were reviewed.[33] Although they do not focus on classroom instruction, these studies relate to teachers' work in their professional lives. Information about these research studies is included in Table 4.

The two studies report similar findings from different contexts that indicate that greater content knowledge tended to be correlated with better formation and functioning of professional communities of mathematics teachers within schools. One study[34] looked at the elementary level and the other[35] was focused at the high school level.

One limitation of both studies is the lack of detailed measures of content knowledge that may be relevant in supporting professional communities. One of the studies[36] selected schools because the level of professional community among the mathematics teachers was known to be very different, so there may be an inherent bias in the sample selection. In the other study[37], the researchers were the ones who provided the professional development intended to deepen the teachers' content knowledge. Although potential research biases were addressed in the study, the likelihood that participants would respond in positive ways to the researchers about the effectiveness of the professional development remains a concern.

Despite these limitations, the two studies are based on thorough qualitative data collection and analyses, and both situate claims about content knowledge as a factor in the results alongside other important factors. Therefore, they offer an intriguing hypothesis for further study and an emerging basis for theory.

---

[33] Lachance & Confrey, 2003; Nickerson & Sowder, 2002.

[34] Nickerson & Sowder, 2002.

[35] Lachance & Confrey, 2003.

[36] Nickerson & Sowder, 2002.

[37] Lachance & Confrey, 2003.

Table 4
Studies Examining the Relationship of Teachers' Disciplinary Mathematics Content Knowledge and Their Professional Community Building

Name of Study	Grade Level	Number of Teachers	Content Strand(s): Number and operations	Content Strand(s): Disciplinary Content	Content Strand(s): Ways of Knowing	Content Strand(s): Pedagogical Content	Type(s) of Knowledge: Algebra	Type(s) of Knowledge: Measurement	Type(s) of Knowledge: Various	Measure(s) of Teacher Content Knowledge: Assessments	Measure(s) of Teacher Content Knowledge: Interviews	Measure(s) of Teacher Content Knowledge: Observations	Measure(s) of Teacher Content Knowledge: Other approach
Interconnecting content and community: A qualitative study of secondary mathematics teachers (Lachance & Confrey, 2003)	9–12	1 school		•					•				•
What factors influence the formation of teachers' professional communities and why should we care (Nickerson & Sowder, 2002)	K–5	2 schools		•					•				•

Teachers' Mathematics Content Knowledge Is Related to Their Students' Learning

Four studies investigating the relationship of teacher mathematics content knowledge and student outcomes were identified. Two were conducted with teachers in the elementary grades,[38] and two with teachers in both the elementary and middle grades,[39] leaving somewhat limited the empirical knowledge base regarding this relationship across the K–12 spectrum. Information about these studies is summarized in Table 5.

In their study of the predictive validity of instruments for measuring teachers' content knowledge, Hill, Rowan, and Ball[40] conducted a large-scale study of the mathematics achievement gains of nearly 3000 first and third grade students in more than 115 schools. To increase the likelihood of substantial variation in teachers' content knowledge, approximately one-fourth of the schools included in this study were selected from those participating in each of three comprehensive school reform programs—America's Choice, Success for All, and Accelerated Schools Project—and the one-fourth that were not participating in such a program were selected to match the other schools in terms of demographics and community and district settings.

Assessments of teachers' mathematics content knowledge, including items focused on disciplinary content and others focused on pedagogical uses of content understanding, were used to predict student achievement. Student achievement was measured each fall and spring over a three-year period, for two cohorts of students—one group entering the study as kindergartners and followed through second grade, the other group entering the study as third graders and followed through fifth grade. The study found that student achievement scores at both grade levels were positively correlated with measures of their teachers' mathematics content knowledge. In addition to accounting for students' prior mathematics knowledge through the longitudinal measures of student achievement, the study included variables for teachers' experience, educational coursework, and certification in order to strengthen the conclusions drawn about the relationship between teachers' content knowledge and their students' achievement in mathematics. The study also controlled for effects of a number of student factors, including student demographics and backgrounds, and tested for potential effects due to student mobility, finding no evidence of bias in the results.

Both Hanssen and Rockoff and colleagues conducted their studies using versions of the same measures of teachers' mathematics content knowledge developed by Hill and colleagues, but appropriate for both elementary and middle grades teachers. Both studies found a significant, positive relationship between teachers' content knowledge and their students' mathematics achievement.

---

[38] Hill, Rowan, & Ball, 2005; Mullens, Murnane, & Willet, 1996.

[39] Hanssen, 2006; Rockoff, Jacob, Kane, & Staiger, 2008.

[40] Hill et al., 2005.

Hanssen's[41] study involved 114 teachers of grades 3–8 from 11 schools participating in the Milwaukee Mathematics Partnership, a project funded by the National Science Foundation through the Math and Science Partnership program. As part of the evaluation of the project, the study examined effects of teachers' education background, teaching experience, and participation in professional development on two factors: teachers' mathematics content knowledge and their classroom teaching practice. Of most importance here, the study also investigated the effect of these two factors on student achievement. Student achievement was measured at the student level using standardized test scores, but to provide an indicator at the teacher level, each student's score was recoded as either proficient or not proficient to determine a percent proficient value for each teacher. This approach to recoding achievement eliminated potentially important variability at the student level, both by restricting individual student scores to a dichotomy and by aggregating student scores within teachers. Statistical testing of this path model revealed only one significant relationship among those tested—a positive association of teacher content knowledge and student achievement.

Rockoff and colleagues[42] focused their study on teachers in their first year of teaching in grades 4–8 in New York City. The study investigated and compared relationships between a number of teacher characteristics (education background, certification and path into the teaching profession, mathematics content knowledge for teaching, and personality traits) and student mathematics achievement. The positive association of teachers' mathematics content knowledge and students' mathematics achievement was one of the few statistically significant relationships with achievement that were identified in the study. (The others were a positive association with national rank of the teachers' college, and higher achievement scores for students of traditionally- certified teachers compared to students of teachers certified through a Teaching Fellows program.) The primary limitation of the study was incomplete information on how the sample was selected, although the discussion of potential biases, including some attention to potential selection bias, is thorough.

The study conducted by Mullens and colleagues[43] examined teacher content knowledge as a predictor of achievement scores from a stratified, random sample of third grade classrooms in Belize during the 1990–91 school year. Background data on the students' teachers, including a measure of teachers' mathematics content knowledge, were used to predict student achievement score gains on a standardized test that was administered to students at the beginning and end of the school year. Achievement scores included two scales, one for basic and one for advanced mathematics concepts. The measure of teacher content knowledge was teachers' level of competence on the mathematics exam they had taken at the end of eighth grade. The analysis of students' performance on the advanced concepts scale showed a significant positive relationship with teachers' mathematics content knowledge. No significant relationship was found for students' gains on the basic concepts scale.

---

[41] Hanssen, 2006.

[42] Rockoff et al., 2008.

[43] Mullens et al., 1996.

The main limitation of this study is the measure of teacher content knowledge that was used. It was not concurrent with the data collection on student achievement, likely at least 10 years old for most of the teachers. There were a substantial number of missing cases, possibly introducing some bias within the sample. The variability on this measure was also fairly limited, so the relationship with student learning could be examined only within a fairly narrow range of teacher knowledge.

Three of the four studies in this group could be characterized as very carefully conducted, controlling for and testing a number of important factors that might account for differences in students' achievement or bias the results.[44] The statistical analyses were thoroughly described and well reasoned to support the findings. The sampling strategies, too, were well described and appropriate. The sizes of the effects of teacher content knowledge on student achievement were meaningful. The findings of these studies were strongly supported, and similar research in other contexts is certainly warranted. The fourth study[45] in this group included careful data collection and a clear model that was tested, but inadequate information on sampling and limited analytic techniques given the data that were available. Generalizability from the samples used in these studies to the populations of teachers they represent in the local contexts of the studies is handled quite well, primarily due to strong sampling plans and careful considerations of non-response and attrition from the studies. From any of these studies, alone, generalization to populations of teachers in other contexts is not strongly supported empirically, but the consistency of findings across studies is striking. Further study of the generalizability of the relationship across the entire K–12 spectrum is warranted.

[44] Hill et al., 2008; Mullens et al., 1996; Rockoff et al., 2008.

[45] Hanssen, 2006.

Table 5
Studies Examining the Relationship of Teachers' Mathematics Content Knowledge and Student Learning

			Content Strand(s)				Type(s) of Knowledge			Measure(s) of Teacher Content Knowledge			
Name of Study	Grade Level	Number of Teachers	Number and operations	Algebra	Geometry/Measurement	Various	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other approach
The Milwaukee Mathematics Partnership: A path model for evaluating teacher and student effects. (Hanssen, 2006)	3-8	114				•	•		•	•			
Effects of teachers' mathematical knowledge for teaching on student achievement (Hill et al., 2005)	K–5	699	•				•		•	•			
The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. (Mullens et al., 1996)	3	72				•	•			•			
Can you recognize an effective teacher when you recruit one? (Rockoff et al., 2008)	4-8	4877				•	•		•	•			

Bibliography for Why Teachers' Mathematics Content Knowledge Matters:
A Summary of Studies

Additional information on how these studies were identified and reviewed, and a summary of the methodology can be found at:

http://www.mspkmd.net/index.php?page=01_4a-3b2

Anders, D. (1995). A teacher's knowledge as classroom script for mathematics instruction. *Elementary School Journal*, *95*(4), 311–324.

Bright, G. W., Bowman, A. H., & Vacc, N. N. (1998, April). *Teachers' frameworks for understanding children's mathematical thinking.* Paper presented at the annual meeting of the American Education Research Association, San Diego, CA.

Cai, J. (2005). U.S. and Chinese teachers' constructing, knowing, and evaluating representations to teach mathematics. *Mathematical Thinking and Learning An International Journal, 7*(2), 135–169.

Chazan, D., Yerushalmy, M., & Leikin, R. (2008). An analytic conception of equation and teachers' views of school algebra. *The Journal of Mathematical Behavior, 27,* 87–100.

Chi-chung, L., Yun-peng, M., & Ngai-ying, W. (1999, March). *Teacher development, not accountability control, is the key to successful curriculum implementation: A case study of two primary schools in northeast China.* Paper presented at the annual meeting of the American Education Research Association, Montreal, Canada.

Chinnappan, M. & Thomas, M. (1999). Conceptual modeling of functions by an experienced teacher. Making the difference. *Proceedings of the Annual Conference of the Mathematics Education Research Group of Australasia Incorporated,* Sydney, Australia.

Fennema, E., Carpenter, T.P., Franke, M.L., Levi, L., Jacobs, V., & Empson, B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403–434.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal, 30*(3), 555–583.

Fernández, E. (1997, March). *The* "standards-*like" role of teachers' mathematical knowledge in responding to unanticipated student observations.* Paper presented at the annual meetings of the American Educational Research Association, Chicago, IL.

Hanssen, C. E. (2006, October). *The Milwaukee Mathematics Partnership: A path model for evaluating teacher and student effects*. Paper presented at the MSP Evaluation Summit II, Minneapolis, MN.

Heaton, R. M. (1992). Who is minding the mathematics content? A case study of a fifth-grade teacher. *The Elementary School Journal, 93*(2), 153–162.

Heid, M. K., Blume, G. W., Zbiek, R. M., & Edwards, B. S. (1999). Factors that influence teachers learning to do interviews to understand students' mathematical understandings. *Educational Studies in Mathematics, 37*, 223–249.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., & Sleep, L.. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406.

Izsak, A. (2008). Mathematical knowledge for teaching fraction multiplication. *Cognition and Instruction, 26*(1), 95–143.

LaChance, A. & Confrey, J. (2003). Interconnecting content and community: A qualitative study of secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*(2), 107–137.

Lehrer, R. & Franke, M. L. (1992). Applying personal construct psychology to the study of teachers' knowledge of fractions. *Journal for Research in Mathematics Education, 23*(3), 223–241.

Leung, F. & Park, K. (2002). Competent students, competent teachers? *International Journal of Educational Research, 37*(2), 113–129.

Llinares, S. (2000). Secondary school mathematics teacher's professional knowledge: A case from the teaching of the concept of function. *Teachers and Teaching: Theory and Practice, 6*(1), 41–62.

Lloyd, G. M. (2002). Reform-oriented curriculum implementation as a context for teacher development: An illustration from one mathematics teacher's experience. *Professional Educator, 24*(2), 51–61.

Lloyd, G. M. & Wilson, M. S. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*(3), 248–274.

Lubinski, C. A. (1993). More effective teaching in mathematics. *School Science and Mathematics, 93*(4), 198–202.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teacher's understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Manouchehri, A. (1998). Mathematics curriculum reform and teachers: What are the dilemmas? *Journal of Teacher Education, 49*(4), 276–286.

Manouchehri, A. & Goodman, T. (1998). Mathematics curriculum reform and teachers: Understanding the connections. *Journal of Educational Research, 92*(1), 27–41.

Manouchehri, A. & Goodman, T. (2000). Implementing mathematics reform: The challenge within. *Educational Studies in Mathematics, 42,* 1–34.

Mullens, J. E., Murnane, R. J., & Willett, J. B. (1996). The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. *Comparative Education Review, 40*(2), 139–157.

Nickerson, S. D. & Sowder, J. T. (2002, July). *What factors influence the formation of teachers' professional communities and why should we care?* Paper presented at the 26th annual meetings of the International Group for the Psychology of Mathematics Education, Norwich, England.

Prawat, R. S. (1992). Are changes in views about mathematics teaching sufficient? The case of a fifth-grade teacher. *The Elementary School Journal, 93*(2), 196–211.

Prawat, R. S., Remillard, J., Putnam, R. T., & Heaton, R. M. (1992). Teaching mathematics for understanding: Case studies of four fifth-grade teachers. *The Elementary School Journal, 93*(2), 145–152.

Putnam, R. T. (1992). Teaching the "hows" of mathematics for everyday life: A case study of a fifth-grade teacher. *The Elementary School Journal, 93*(2), 163–177.

Putnam, R.T., Heaton, R.M., Prawat, R.S., & Remillard, J.T. (1992). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. *The Elementary School Journal, 93*(2), 213–228.

Remillard, J. (1992). Teaching mathematics for understanding: A fifth-grade teacher's interpretation of policy. *The Elementary School Journal, 93*(2), 189–193.

Rockoff, J. E., Jacob, B. A., Kane, T. J., & Staiger, D. O. (2008). *Can you recognize an effective teacher when you recruit one?* NBER Working Paper No. 14485. Cambridge, MA: National Bureau of Economic Research.

Schwartz, J. E. & Riedesel, C. A. (1994). *The relationship between teachers' knowledge and beliefs and the teaching of elementary mathematics.* Paper presented at the annual meetings of the American Association of Colleges for Teacher Education, Chicago, IL.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119–150.

Sowder, J. T., Phillip, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction*. Albany, NY: State University of New York Press.

Spillane, J. P. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *The Elementary School Journal*, *100*(4), 307–330.

Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject-matter knowledge and elementary instruction: A case from functions and graphing. *American Educational Research Journal*, *27*(4), 639–663.

Thompson, A. G. & Thompson, P. W. (1996). Talking about rates conceptually, Part II: Mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 27*(1), 2–24

Thompson, P. W. & Thompson, A. G. (1994). Talking about rates conceptually, Part I: Teacher's struggle. *Journal for Research in Mathematics Education*, *25*(3), 279–303.

Vale, C. & McAndrew, A. (2008, June). *Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications.* Paper presented at the Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane, Queensland, Australia.

Warfield, J. (2001). Teaching kindergarten children to solve word problems. *Early Childhood Education Journal*, *28*(3), 161–167.

Wilkins, J. L. M. (2002). The impact of teachers' content knowledge and attitudes on instructional beliefs and practices. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 24,* 10, Athens, GA.

Wilson, M. R. (1994, March). *Implications for teaching of one middle school mathematics teacher's understanding of fractions.* Paper presented at the annual meeting of the American Education Research Association, New Orleans, LA.

Zbiek, R. M. (1995). Her math, their math: An in-service teacher's growing understanding of mathematics and technology and her secondary students' algebra experience. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 17,* 9, Columbus, OH.